Math 184A First Exam 22 October 2003

- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK, but you may have a page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (20 pts.) Consider the three-digit numbers that do not begin with zero and also have all digits distinct. For example, 342, 901, and 123 are allowed but 034, 122 and 474 are not allowed.

   (a) How many are there?

   (b) How many have the sum of their digits odd? (For example, 342 and 126 have odd sums.)

   Hint: You might consider cases depending on which digits are odd and which are even.

2. (10 pts.) Consider the strictly decreasing functions from $\{1,2,3\}$ to $\{1,2,\ldots,99\}$ ordered lexicographically. (This is the usual ordering.) What is the rank of the function whose one-line form is 6,3,1?

$$\binom{2}{1}=2 \quad \binom{3}{1}=\binom{3}{2}=3 \quad \binom{4}{1}=\binom{4}{3}=4 \quad \binom{4}{2}=6$$

$$\binom{5}{1}=5 \quad \binom{5}{2}=\binom{5}{3}=10 \quad \binom{6}{1}=6 \quad \binom{6}{2}=15 \quad \binom{6}{3}=20$$

3. (10 pts.) Let $P(n,k)$ be the number of $(2k+1)$ long sequences

$$\underbrace{a_0<a_1<\cdots<a_{k-1}}_{k\ \text{items}}<a_k>\underbrace{a_{k+1}>\cdots>a_{2k}}_{k\ \text{items}}$$

where all the a_i are in $\{1,2,\ldots,n\}$. For example, the 10 sequences counted by $P(4,2)$ include

$$1,2,3,2,1 \qquad 1,2,4,2,1 \qquad 1,2,4,3,1 \qquad 2,3,4,2,1$$

Obtain a formula for $P(n,k)$. It will probably be a sum involving binomial coefficients.

Hint: How many sequences have $a_k=t$?

To receive credit, you must explain clearly why your formula is corect.

END OF EXAM