1. (a) The graph is a triangle a, b, c with two additional edges attached at a.
   (b) There are three spanning trees. Each tree is obtained by removing exactly one of the edges $\{a, b\}$, $\{a, c\}$. and $\{b, c\}$.
   (c) The tree obtained by removing $\{b, c\}$ is *not* lineal. The other two trees are.
   (d) This can be done in various ways. Here's one. Color a (x ways), then color c, d and e ($x-1$ ways each since the only constraint is that they differ from the color of a). Finally, color b ($x-2$ ways since it must differ from both a and c). The answer is $x(x-1)^3(x-2)$.

2. We have $G_0 = G_1 = x$ and $G_{11} = x^2$. Thus $G_{0^*} = \frac{1}{1-x}$ and $G_{(11)^*} = \frac{1}{1-x^2}$. Hence

$$G_{00^*} = \frac{x}{1-x}, \quad G_{00^*(11)^*1} = \frac{x}{1-x}\frac{x}{1-x^2}, \quad G_{(00^*(11)^*1)^*} = \frac{1}{1-\frac{x}{1-x}\frac{x}{1-x^2}}$$

and so

$$A(x) = \frac{1}{1-\frac{x}{1-x}\frac{x}{1-x^2}}\frac{x}{1-x} = \frac{(1-x^2)x}{(1-x)(1-x^2)-x^2} = \frac{(1-x^2)x}{1-x-2x^2+x^3}.$$

Thus $P(x) = x(1-x^2)$.

3. Multiply both sides of (1) by the denominator $1-x-2x^2+x^3$ and find the coefficient of x^n on both sides. Since $P(x)$ is a cubic, we have

$$a_n - a_{n-1} - 2a_{n-2} + a_{n-3} = 0$$

for $n > 3$. Rearranging gives the recursion $a_n = a_{n-1} + 2a_{n-2} - a_{n-3}$.

4. This can be done using Principle 11.6 or Example 11.27. The singularity closest to the origin is the smallest root of the denominator of $A(x)$, namely β. We have $A(x) = (1-x/\beta)^{-1}g(x)$ where

$$g(x) = \frac{P(x)}{-\beta(x-\alpha)(x-\gamma)} \quad \text{and} \quad g(\beta) = \frac{P(\beta)}{\beta(\beta-\alpha)(\gamma-\beta)}.$$

Thus $A = g(\beta)$, $B = 0$ and $C = 1/\beta$.

5. This type of problem was discussed in Example 10.9 and in class. The answer is

$$\frac{[x^n]\, A_y(x,1)}{[x^n]\, A(x,1)}.$$