

1. (a) $9 \times 9 \times 8$ since the first digit must not be zero, the second anything except the first, and the third anything but the first two.
- (b) With O for odd and E for even, the four possible patterns and the number of 3-digit numbers having each pattern are

$$EEO : 4 \times 4 \times 5 \quad EOO : 4 \times 5 \times 4 \quad OEO : 5 \times 5 \times 4 \quad OOO : 5 \times 4 \times 3.$$

The answer is the sum of these. By the way, this is 40×8 , somewhat less than half of (a), which is 81×8 .

2. Use the formula on page 65 or, if you don't remember it, draw the relevant portion of the decision tree.

(a) $\text{RANK}(7, 3, 1) = \binom{6}{3} + \binom{2}{2} + \binom{0}{1} = 21.$

- (b) We use the greedy algorithm method to compute $\text{UNRANK}(17)$.

- Since $\binom{6}{3} = 20$ and $\binom{5}{3} = 10$, $f(1) = 6$ and we have $17 - 10 = 7$ left to go.
- Since $\binom{5}{2} = 10$ and $\binom{4}{2} = 6$, $f(2) = 5$ and we have $7 - 6 = 1$ left to go.
- Since $\binom{1}{1} = 1$, $f(3) = 2$.

Thus the function is 6,5,2 in one-line form.

5. Either

- (0) n does not appear AND the remaining $n - 1$ elements form a k -list, giving the term $1 \times L(n - 1, k)$, OR
- (1) n appears in one of the k positions AND the remaining $n - 1$ elements form a $(k - 1)$ -list in the remaining $k - 1$ positions, giving $k \times L(n - 1, k - 1)$, OR
- (2) n appears in two of the k positions AND the remaining $n - 1$ elements form a $(k - 2)$ -list in the remaining $k - 1$ positions, giving $\binom{k}{2} \times L(n - 1, k - 2)$.

In other words, $a = 1$, $b = k$ and $c = \binom{k}{2}$.