- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK, but you may have a page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (20 pts.) Consider the three-digit numbers that do not begin with zero and also have all digits distinct. For example, 342, 901, and 123 are allowed but 034, 122 and 474 are not allowed.

   (a) How many are there?

   (b) How many of them are odd?

   Hint: Consider cases depending on which digits are odd and which are even.

2. (20 pts.) Consider the strictly decreasing functions from $\{1,2,3\}$ to $\{1,2,\ldots,99\}$ ordered lexicographically. (This is the usual ordering.)

   (a) What is the rank of the function whose one-line form is 7,3,1?

   (b) Which function has rank 17?

$$\binom{2}{1} = 2 \quad \binom{3}{1} = \binom{3}{2} = 3 \quad \binom{4}{1} = \binom{4}{3} = 4 \quad \binom{4}{2} = 6$$

$$\binom{5}{1} = 5 \quad \binom{5}{2} = \binom{5}{3} = 10 \quad \binom{6}{1} = 6 \quad \binom{6}{2} = 15 \quad \binom{6}{3} = 20$$

3. (10 pts.) Let $L(n,k)$ be the number of (ordered) k-element lists that can be formed from the set $S = \{1,2,\ldots,n\}$ with the restriction that **no element of S can appear more than twice in a list**. (If I'd said "more than once", it would have been lists without repeats.)

   By considering where n appears in a list obtain a recursion of the form

$$L(n,k) = aL(n-1,k) + bL(n-1,k-1) + cL(n-1,k-2)$$

   where a, b and c may be constants or simple functions of k.

END OF EXAM