- Please put your name and ID number on your exam. If you are not using a blue book, put your name on every page.
- The exam is CLOSED BOOK.
- Calculators are allowed.
- **You must show your work to receive credit.**

1. For each of the following,
   EITHER give an example of the thing that is described
   OR explain why none exists.
   (a) A surjection from $\{1,2,3\}$ to $\{a,b,c,d\}$.
   (b) An injection from $\{1,2,3\}$ to $\{a,b,c,d\}$.
   (c) A permutation f of $\{1,2,3,4\}$ such that f^{12} is NOT the identity function. The identity function is the function g such that $g(x)=x$ for all x.
   Remember that $f^{12}(x)$ is $f(f(\cdots f(x)))$, not $(f(x))^{12}$.
   (d) An involution f of $\{1,2,3,4,5,6\}$ with exactly 4 cycles.

2. How many 6 card hands contain 3 pairs?
   Important: Give a careful justification of how you found your answer, not just a bunch of numbers multiplied together without explanation.

3. A magazine article lists 8 yes/no questions. Each question must be answered "yes," "no" or "don't know". The article says that a person is credulous if he or she answers at least 6 of the questions with a "yes". How many ways are there to answer the questions so that you receive the label "credulous?"

4. Let S be an n-set. It was shown in the text (and in class) that the number of subsets that can be formed from S is 2^n.

   Generalize this: State and prove a formula for the number of multisets that can be formed from S where *each element is repeated at most k times.*
   When $k=1$, your formula should become 2^n.

5. Let $S(n,k)$ be the Stirling numbers of the second kind; that is, $S(n,k)$ is the number of ways to partition an n-set into k unordered blocks. Prove that

$$S(n,k)=\sum_{j=1}^{n}\binom{n-1}{j-1}S(n-j,k-1) \text{ for } n>0 \text{ and } k>0.$$

END OF EXAM