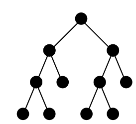
Math 184A Second Exam 19 November 2001

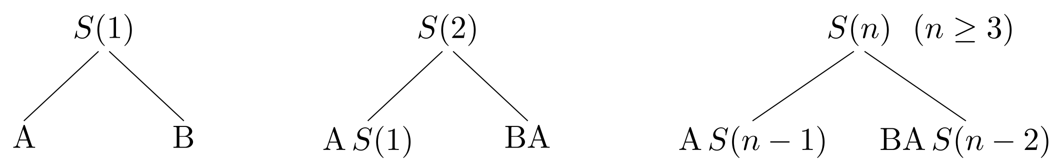
- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK, but you may have a page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. Let $G = (V, E)$ where $V = \{0, 1, a, b, A, B\}$ and
   $E = \big\{\{0,1\},\ \{0,a\},\ \{0,b\},\ \{0,A\},\ \{0,B\},\ \ \{a,b\},\ \{A,B\}\big\}.$
   Sketch the simple graph G and
   compute its chromatic polynomial.

2. **Compute** the rank of the binary RP-tree shown here. For your information, $b_1 = b_2 = 1$, $b_3 = 2$, $b_4 = 5$, $b_5 = 14$, $b_6 = 42$, and $b_7 = 132$.



3. The local description of a decision tree for constructing sequences of A's and B's is given below. The notation BA $S(n-2)$ means place BA in front of each sequence produced by $S(n-2)$.

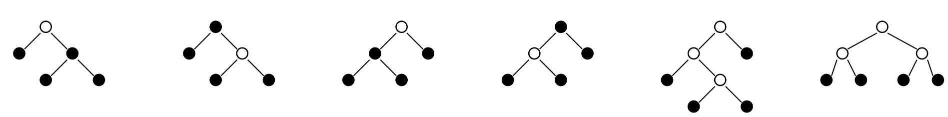


   Let $S^*(n)$ denote the entire decision tree. Thus $S^*(1) = S(1)$ and $S^*(2)$ has the three leaves AA, AB, and BA.

   (a) **Find** a recursion for s_n, the number of leaves of $S^*(n)$.
   Remember to include initial conditions.

   (b) **Prove** that the leaves of $S^*(n)$ are sequences of length n and that their order from left to right is alphabetic.

4. A binary RP-tree has information stored at each leaf vertex. Each non-leaf vertex may or may not have information stored at it. Let t_n be the number of such trees with information stored at exactly n vertices and let $T(x) = \sum_{n=1}^{\infty} t_n x^n$ be the generating function. The following picture shows some of the nine trees that contribute to t_4. An empty circle indicates a vertex with no information.



   Find a formula for $T(x)$ similar to the formula $B(x) = x + B(x)^2$ we found for binary RP-trees. *To receive credit you must justify your formula; that is, explain how you got it.*

END OF EXAM