Math 184A First Exam 15 October 2001

- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK, but you may have a page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. Let A, B and C be finite sets. Suppose $f : A \to B$ and $g : B \to C$ are functions. Define $h : A \to C$ by $h(x) = g(f(x))$.
   (a) Prove or give a counterexample:
   If h is a surjection (an onto function), then f must be a surjection.
   (b) Prove or give a counterexample:
   If h is a surjection (an onto function), then g must be a surjection.

2. A 5 person committee is to be chosen from a set of 6 men and 7 women.
   (a) How many possible committees are there?
   (b) If the committee must contain at least 2 men and at least 2 women, how many possible committees are there?

3. Prove that exactly half of the 2^{2n-1} compositions of $2n$ contain at most n parts.
   For example, when $n = 2$ the compositions of 4 with at most 2 parts are

$$4 \qquad 3+1 \qquad 2+2 \qquad 1+3.$$

   Warning: It was proved in a homework exercise that the average number of parts is $(2n+1)/2$, but you cannot do the problem just by knowing the average number of parts. For example, the compositions in the set $\{2+2,\ 3+1,\ 1+3,\ 1+1+1+1\}$ have an average of $(2n+1)/2$ parts but 3/4 of the compositions in the set have at most 2 parts.

4. We want to count 4-bead necklaces that can be made using a supply of $k > 4$ different types of beads. (We allow rotations of a necklace, but not flipping over.)
   (a) How many necklaces can be made if each type of bead can be used as often as you wish?
   (b) How many necklaces can be made if each type of bead can be used at most once in each necklace?

END OF EXAM