Name ______________________________ ID No. ______________

There are 225 points possible.

1. (30 pts.) Answer briefly in English using a minimum of mathematics.
   (a) What is the Church-Turing thesis regarding Turing machines?

   (b) How do certificates and verifiers relate the class NP to ordinary Turing machines?

   (c) What does "M accepts the string w" mean when M is a nondeterministic automaton or Turing machine?

2. (30 pts.) Write regular expressions for the following when $\Sigma = \{0, 1\}$.
   (a) $\overline{(\Sigma^*1)} \cap \overline{(1\Sigma^*)}$.
   Hint: First describe the strings in the language without using "not."

   (b) $\{w \mid w$ has an even number of 0's, or 1's, or both$\}$.
   (For example, ϵ, 010, 110, and 1010 are in the language, but 01 is not.)

3. (45 pts.) Beware of guessing:

correct answer +5pts. incorrect answer −3pts. no answer 0pts

- ____ A nondeterministic Turing machine can recognize more languages than a standard Turing machine.
- ____ Context free grammars can generate languages that DFAs cannot recognize.
- ____ Context free grammars can generate languages that Turing machines cannot recognize.
- ____ $\{a^k b^k \mid 0 \leq k \leq 5\}$ is a regular language.
- ____ There are polynomial time algorithms for some NP-complete problems.
- ____ If L is an NP-complete language and M is polynomial-time reducible to L, then M is also an NP-complete language.
- ____ The class of regular languages is closed under complementation.
- ____ The class of context-free languages is closed under complementation.
- ____ The class of Turing-recognizable languages is closed under complementation.

4. (30 pts.) Construct CFGs that generate the following languages when $\Sigma = \{0, 1\}$.

(a) $\{ww^{\mathcal{R}} \mid w \in \Sigma^*\}$, where $w^{\mathcal{R}}$ is the reverse of the string w.

(b) $\{0^i 1^j 0^k \mid \text{where } i + j = k\}$.

5. (30 pts.) Construct PDAs that recognize the following languages when $\Sigma = \{0, 1\}$.
   (a) $\{w \mid w$ contains at least two 1's$\}$.

   (b) $\{0^i 1^j 0^k \mid \text{where } i + k = j\}$. *Warning*: This is *not* the same language as in 4(b).

6. (30 pts.) NEQ_{CFG} is the set of pairs G_1, G_2 of CFGs such that G_1 and G_2 generate different languages. Prove that NEQ_{CFG} is Turing-recognizable. That is, prove that you can build a Turing machine that will take two CFGs and accept them if and only if they produce different languages.
   Remark: To "build a Turing machine," it is sufficient to give a high level description of an algorithm — you need not give details such as state transitions and tape reading/writing.
   Hint: Since CFGs can be put in Chomsky normal form, assume that G_1 and G_2 are in Chomsky normal form.

7. (30 pts.) Prove that P (the class of languages decidable in polynomial time) is closed under complementation and union.