

3. (45 pts.) Beware of guessing:

correct answer +5pts. incorrect answer -3pts. no answer 0pts

- ___ A nondeterministic Turing machine can recognize more languages than a standard Turing machine.
 - ___ Context free grammars can generate languages that DFAs cannot recognize.
 - ___ Context free grammars can generate languages that Turing machines cannot recognize.
 - ___ $\{a^k b^k \mid 0 \leq k \leq 5\}$ is a regular language.
 - ___ There are polynomial time algorithms for some NP-complete problems.
 - ___ If L is an NP-complete language and M is polynomial-time reducible to L , then M is also an NP-complete language.
 - ___ The class of regular languages is closed under complementation.
 - ___ The class of context-free languages is closed under complementation.
 - ___ The class of Turing-recognizable languages is closed under complementation.
4. (30 pts.) Construct CFGs that generate the following languages when $\Sigma = \{0, 1\}$.
- (a) $\{ww^{\mathcal{R}} \mid w \in \Sigma^*\}$, where $w^{\mathcal{R}}$ is the reverse of the string w .

 - (b) $\{0^i 1^j 0^k \mid \text{where } i + j = k\}$.

5. (30 pts.) Construct PDAs that recognize the following languages when $\Sigma = \{0, 1\}$.
- (a) $\{w \mid w \text{ contains at least two 1's}\}$.

(b) $\{0^i 1^j 0^k \mid \text{where } i + k = j\}$. *Warning:* This is *not* the same language as in 4(b).

6. (30 pts.) NEQ_{CFG} is the set of pairs G_1, G_2 of CFGs such that G_1 and G_2 generate different languages. Prove that NEQ_{CFG} is Turing-recognizable. That is, prove that you can build a Turing machine that will take two CFGs and accept them if and only if they produce different languages.

Remark: To “build a Turing machine,” it is sufficient to give a high level description of an algorithm — you need not give details such as state transitions and tape reading/writing.

Hint: Since CFGs can be put in Chomsky normal form, assume that G_1 and G_2 are in Chomsky normal form.

7. (30 pts.) Prove that P (the class of languages decidable in polynomial time) is closed under complementation and union.