Name ______________________________ ID No. ______________

There are 200 points possible.

1. (40 pts.) Indicate whether true or false. Beware of guessing:

   correct answer +5pts. incorrect answer −3pts. no answer 0pts

   (a) ____ If L is a regular language, then $\overline{L}$ must be a regular language.

   (b) ____ If L is a context free language, then $\overline{L}$ must be a context free language.

   (c) ____ If L is decidable, then $\overline{L}$ must be decidable.

   (d) ____ If L is Turing recognizable, then $\overline{L}$ must be Turing recognizable.

   (e) ____ Personal computers can recognize languages that Turing machines cannot.

   (f) ____ It is possible to build a "universal simulator"; that is, a Turing machine that, when given the description and input for any Turing machine, will be able to simulate that Turing machine on that input.

   (g) ____ A language L in NP is "NP-complete" if every other language in NP is polynomial time reducible to L.

   (h) ____ We can build a Turing machine that takes as input the description of a 2-stack PDA plus the input to the PDA and decides if the PDA will stop.

2. (45 pts.) Do each of the following or explain why you cannot. If you give an example, explain why it is correct.

   (a) Two regular languages whose intersection is *not* regular.

   (b) Two context free languages whose intersection is *not* context free.

   (c) A language which is in NP but is *not* in P.

3. (45 pts.) Let the alphabet be $\Sigma = \{0, 1\}$.
   - Let A be set of strings in Σ^* which contain no adjacent ones.
   - Let B be the set of strings in Σ^* which contain an even number of zeros.
   - Let $C = (A \cap \overline{B}) \cup (\overline{A} \cap B)$.

   For example, 011000 and 100101 are in C but 01100 and 0100101 are not in C.

   (a) Write a regular expression for A.

   (b) Prove that B is a regular language.

   (c) Prove that C is a regular language.
   HINT: You may use the results in (a) and (b) even if you have not done those parts.

4. (20 pts.) Construct a context free grammar for the language generated by the regular expression $(a^* \cup b) \circ (ba^*)^*$.

5. (20 pts.) Suppose L and M are Turing-recognizable languages. Prove that $L \cup M$ and $L \cap M$ are Turing-recognizable.

6. (30 pts.) Recall that EQ_{CFG} is the set of pairs G_1, G_2 of CFGs such that G_1 and G_2 generate the same language. It was remarked on page 158 that EQ_{CFG} is *not* decidable.

   (a) Let $EQ_{\mathrm{CFG,n}}$ be the set of pairs G_1, G_2 of CFGs such that the languages generated by G_1 and G_2 contain exactly the same strings of length n. Prove that $EQ_{\mathrm{CFG,n}}$ is decidable. **HINT**: Use Chomsky normal form.

   (b) What is wrong with the following proof that EQ_{CFG} is Turing decidable?

   For each n, run $EQ_{\mathrm{CFG,n}}$. If $EQ_{\mathrm{CFG,n}}$ stops with `reject` *for some n, then* `reject`*; otherwise,* `accept`.