There are 125 points total. (At 5 pts. = 1%, the first exam is 20% and this is 25%.)

1. (40 pts.) Indicate whether true or false. Beware of guessing:

   correct answer +5pts. incorrect answer −3pts. no answer 0pts

   (a) T Every finite set of strings is a regular language.
   (b) F If L is a Turing-recognizable language, so is $\overline{L}$.
   (c) F There are CFLs that Turing machines cannot recognize.
   (d) F A nondeterministic Turing machine can recognize more languages than a standard Turing machine can.
   (e) T A 2-stack PDA can recognize more languages than a standard 1-stack PDA can.
   (f) F A 2-tape Turing machine can recognize more languages than a standard 1-tape Turing machine can.
   (g) F There exists a Turing machine that can decide if two context free grammars generate the same language.
   (h) T There exists a Turing machine which can decide if two DFAs recognize the same language.

2. (25 pts.) Prove: If L is decidable, then $L^{\mathcal{R}}$ is decidable
   Hint: Make use of the Turing machine that decides L.

   Ans. Suppose T_L decides L. Let M be a Turing machine that reverses its input and feeds the result to T_L. The machine M decides $L^{\mathcal{R}}$.

3. (36 pts.) Give an example of a language which satisfies each of the following. If it is in the text (including Exercises and Problems), or is a simple modification of one of these, no proof is needed. Otherwise, give a proof.

   (a) A CFL which is NOT regular.

   (b) A decidable language which is NOT a CFL.

   (c) A Turing-recognizable language which is NOT decidable.

   Ans. There is more than one possible answer to these questions.
   Here are some possibilities.
   (a) $\{0^n1^n \mid n \geq 0\}$ or $\{ww^{\mathcal{R}} \mid w \in \{0,1\}^*\}$
   (b) $\{0^n1^n0^n \mid n \geq 0\}$ or $\{ww \mid w \in \{0,1\}^*\}$
   (c) A_{TM}

4. (24 pts.) Let $L = \{ww^{\mathcal{R}}|w \in \{0,1\}^*\}$. Construct *either*

   (i) a context free grammar to generate L *or*

   Ans. A simple one is $S \to \epsilon \mid 0S0 \mid 1S1$

   (ii) a PDA to recognize L.

   Ans. There are 4 states q_0, q_w, $q_{\mathcal{R}}$, q_a, with q_0 the start state and q_a the accept state. The rules are
   1. From q_0, push \$ and go to q_w.
   2. From q_w either go to $q_{\mathcal{R}}$ or push the next input symbol.
   3. From $q_{\mathcal{R}}$ either
      (a) pop \$ and go to q_a, or
      (b) if the stack agrees with the next input symbol, pop it and go to $q_{\mathcal{R}}$.

END