1. (40 pts.) Indicate whether true or false.
   (a) T $\emptyset \subseteq \mathcal{P}(\{0,1\})$. (Recall that $\mathcal{P}(S)$ is the set of all subsets of S.)
   (b) T $\emptyset \in \mathcal{P}(\{0,1\})$.
   (c) T If Σ is an alphabet, $\epsilon \in \Sigma^*$.
   (d) T For any language L, $L \circ L \subseteq L^*$.
   (e) T If $S \subset \Sigma$ is a finite set, then S is a regular language. [Any finite subset of Σ^* is a regular language.]
   (f) F If R_1 and R_2 are regular expressions, then $R_1 \cap R_2$ is a regular expression. [It can be *rewritten* as a regular expression since it describes a regular language, but it is not a regular expression.]
   (g) T If R_1 and R_2 are regular expressions, then there is a regular expression that describes the same language as $R_1 \cap R_2^{\mathcal{R}}$. (Recall that $S^{\mathcal{R}}$ is the reverse of S.)
   (h) T If L_1 and L_2 are regular languages, then $L_1 \circ \overline{L_2}$ is a regular language.

2. (20 pts.) Let $L = (0^*1^*) \cup (01)^*$.
   (a) Indicate which of the following strings are in L and which are not in L

0011	1100	00	11	1010
yes	no	yes	yes	no

   (b) Construct an NFA to recognize the language L.
   You probably have a diagram. Here is an NFA in tabular form:

$$Q = \{q_0, q_1, q_2, q_3\}, \qquad F = \{q_2\}$$

and δ is given by

	ϵ	0	1
q_0	$\{q_1\}$	$\{q_0\}$	$\{q_1\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_1\}$
q_2	$\emptyset$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_2\}$	$\emptyset$

3. (20 pts.) Either give an example of each of the following or explain why no example exists.

   (a) A language that is *not* recognized by any DFA but is recognized by some NFA.

   ANS. This is impossible since the languages recognized by DFAs are the same as those recognized by NFAs, namely the regular languages.

   (b) A language that is *not* regular.

   ANS. Some examples were given in the text. Any of those will do. For example, $\{0^n1^n \mid n \geq 0\}$.

4. (20 pts.) If L is a language, define $\text{tail}\,(L) = \{x \mid wx \in L \text{ for some string } w\}$. For example,

$$\text{tail}\left(\{011, 101\}\right) = \{\epsilon, 1, 01, 11, 011, 101\}.$$

Prove the following: If L is regular, then $\text{tail}\,(L)$ is regular.
Hint: Use NFAs.

ANS. Let M be an NFA that recognizes L and let q_0 be its start state. We may assume that every state in M is reachable from the start state by some path for, if a state is unreachable, we can simply remove it and the edges in and out of it. [This point is needed in the next paragraph. You will not lose credit if you miss it.] Define a new NFA M' that is the same as M except it has a new state q_0' that is its start state and there is an ϵ-edge from q_0' to every state of M.

Suppose s' is accepted by M'. Suppose a path through M' that accepts s' begins with the transition from q_0' to q. Let t be a string that allows M to move from state q_0 to state q. Note that M accepts ts'. We have proved that all strings accepted by M' are in $\text{tail}\,(L)$.

Now suppose that $s' \in \text{tail}\,(L)$. There is some string t such that $ts' \in L$. Since L can follow an ϵ-edge to reach any state M can reach by processing t, it follows that M' accepts s'.

Combining the results of the last two paragraphs, we see that the language recognized by M' is $\text{tail}\,(L)$. Hence $\text{tail}\,(L)$ is regular.