

You may leave sums, products, factorials, binomial coefficients, and so on in your answers.

Q1. A family has 3 girls and 3 boys. *Give reasons for your answers.*

- How many ways can they sit around a circular table?
- How many ways can they sit in a row if girls and boys must alternate; that is, two boys cannot sit next to each other and two girls cannot sit next to each other?

Q2. Homework problems CL-3.1 (3 pair in 6 cards) and CL-3.2 (straight).

Q3. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$. A function is chosen uniformly at random from B^A .

- What is the probability that it is an injection?
- What is the probability that it is a surjection?

To receive credit, be sure to show how you got your answers.

Q4. Here is a permutation f in cycle form: $(1, 6, 3, 2)(4, 7, 5)$.

- Write f in one-line form.
- Write f^{-1} in cycle form.
- Write f^{13} in cycle form.

Q5. Given that

$$E(X) = 5, \quad E(Y) = 0, \quad \text{var}(X) = 2, \quad \text{var}(Y) = 1, \quad \text{cov}(X, Y) = -1.$$

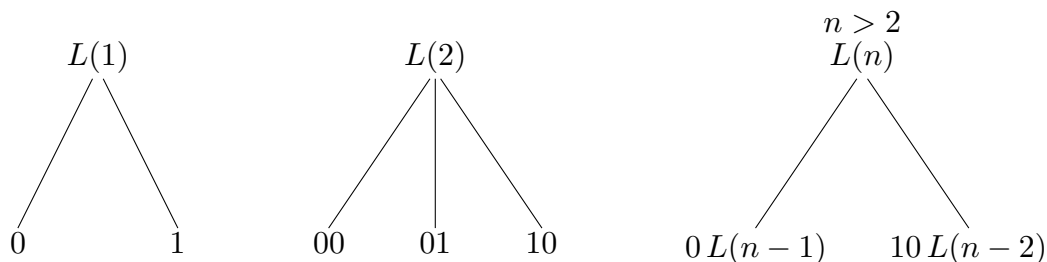
Compute $E(X^2)$ and $\text{var}(X + 3Y)$.

Q6. Draw the decision tree to list all sequences of length six of A's and B's that satisfy the following conditions:

- There are no two adjacent A's.
- There are never three B's adjacent.
- The leaves are in alphabetical order.

Q7. The local description of a decision tree for constructing sequences of 0's and 1's is given below. The notation $10L(n-2)$ means place 10 (1 and 0) in front of each sequence produced by $L(n-2)$.

- Draw the entire decision tree for $n = 3$. How many leaves does it have?
- Let a_n be the number of leaves in the entire decision tree for $L(n)$. Obtain a recursion for a_n . *Remember to include initial conditions.*



- Q8. An urn contains one red ball and one blue ball. I do the following steps:
1. Flip a fair coin. If heads, add a red ball to the urn. If tails, add no balls to the urn.
 2. Remove a ball at random from the urn and replace it with a ball of the opposite color.
 3. Remove a ball at random from the urn.
- Draw the decision tree and use it to answer the following:
- (a) What is the probability that the ball removed in Step 3 is blue?
 - (b) If the ball in Step 3 is blue, what is the probability that the coin toss was heads?
- Q9. Give a graph satisfying the conditions for each problem **OR** explain why none exists.
- (a) A **simple** graph with 4 vertices and 7 edges.
 - (b) A **simple** graph with 4 vertices, 2 edges, and 4 connected components.
 - (c) A **simple** graph with 6 vertices, 6 edges, and 2 cycles.