1. Give an example of each of the following or explain why it cannot be done.
   (a) A bijection from $\{1, 2, 3, 4\}$ to $\{a, b, c\}$.
   A. Impossible. A bijection has it's range and domain the same size.
   (b) A permutation of $\{1, 2, 3, 4, 5, 6, 7\}$ that has a cycle of length 4 and also has a cycle of length 5.
   A. Impossible. The sum of cycle lengths is the size of the set being permuted and $4 + 5 > 7$.

2. A committee contains 7 women and 6 men. We want to form a subcommittee with 5 of these people.
   (a) How many ways can this be done?
   A. $\binom{7+6}{5}$
   (b) How many ways can this be done if the subcommittee must contain at least 2 women and at least 2 men?
   A. There are either (2 men AND 3 women) OR (3 men AND 2 women). By the Rules of Sum and Product, the answer is $\binom{6}{2} \times \binom{7}{3} + \binom{6}{3} \times \binom{7}{2}$.
   Choosing two of each sex and then a fifth committee member via $\binom{6}{2}\binom{7}{2}\binom{9}{1}$ overcounts. For example, if there are 3 women on the committee, the committee is counted 3 times depending on which woman is selected as the fifth committee member.

3. The table below gives the joint distribution function, $h_{X,Y}$, for two random variables X and Y.
   (a) Find the distribution functions f_X for X and f_Y for Y.
   A. $f_X(-1) = f_X(0) = f_X(1) = 1/3$. The distribution function for Y is the same.
   (b) Are X and Y independent? (You must give a correct reason for your answer.)
   A. No. It suffices to give two values r and s for which $P(X = r \& Y = s)$ does not equal $P(X = r)\, P(Y = s)$. Any choices of r and s from $\{-1, 0, 1\}$ have this property.

   Note: Recall that, if X and Y are independent, then $\text{cov}(X, Y) = 0$. This table shows that the converse is false: If you do the calculations, you will find that $\text{cov}(X, Y) = 0$, but we just showed that X and Y are *not* independent.

$h_{X,Y}$	Y=−1	Y=0	Y=+1
X=−1	1/6	0	1/6
X=0	0	1/3	0
X=+1	1/6	0	1/6

4. A deck of cards has 52 cards and 13 of these cards are spades.
   (a) I take seven cards at random from the deck. What is the probability that I get exactly three spades?
   A. We have a set of size 52 and 13 are "bad" (spades). The probability that a subset of size 7 contains exactly 3 bad is $\binom{52-13}{7-3}\binom{13}{3}\Big/\binom{52}{7}$ by the hypergeometric probability formula.
   (b) I take a card at random from the deck, note whether it is a spade, and put it back. If I do this seven times, what is the probability that I get a spade exactly three times?
   A. This is a sequence of 7 independent trials. The probability of spade being drawn is $13/52 = 1/4$. By the binomial distribution, the probability of drawing exactly 3 spades is $\binom{7}{3}(1/4)^3(3/4)^4$.
   You could have left $1/4$ and $3/4$ as unreduced fractions and switched the roles of spade and non-spade; for example, your answer might have been
   $\binom{7}{4}((52-13)/52)^4(1-(52-13)/52)^3$.

   Suggestion: Think of spade and non-spade like bad and good.

END OF EXAM