

- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK, but a PAGE OF NOTES IS ALLOWED.
- Calculators are NOT ALLOWED. You need not evaluate binomial coefficients.
- **You must show your work to receive credit.**

1. Give an example of each of the following or explain why it cannot be done.
  - (a) A bijection from  $\{1, 2, 3, 4\}$  to  $\{a, b, c\}$ .
  - (b) A permutation of  $\{1, 2, 3, 4, 5, 6, 7\}$  that has a cycle of length 4 and also has a cycle of length 5.
  
2. A committee contains 7 women and 6 men. We want to form a subcommittee with 5 of these people.
  - (a) How many ways can this be done?
  - (b) How many ways can this be done if the subcommittee must contain at least 2 women and at least 2 men?
  
3. The table below gives the joint distribution function,  $h_{X,Y}$ , for two random variables  $X$  and  $Y$ .
  - (a) Find the distribution functions  $f_X$  for  $X$  and  $f_Y$  for  $Y$ .
  - (b) Are  $X$  and  $Y$  independent? (You must give a correct reason for your answer.)

$h_{X,Y}$	$Y=-1$	$Y=0$	$Y=+1$
$X=-1$	1/6	0	1/6
$X=0$	0	1/3	0
$X=+1$	1/6	0	1/6

4. A deck of cards has 52 cards and 13 of these cards are spades.
  - (a) I take seven cards at random from the deck. What is the probability that I get exactly three spades?
  - (b) I take a card at random from the deck, note whether it is a spade, and put it back. If I do this seven times, what is the probability that I get a spade exactly three times?

*Suggestion:* Think of spade and non-spade like bad and good.

**END OF EXAM**