Q1. A family has 4 girls and 3 boys.
(a) How many ways can they sit in a row?
(b) How many ways can they sit in a row if boys and girls must alternate?

Q2. How many ways can t teams each of size s be made from st people? The teams have no names or other distinguishing features. Three versions were given depending on student ID number:

$$s = 2, \quad t = 4; \qquad\qquad s = 3, \quad t = 3; \qquad\qquad s = 4, \quad t = 2.$$

Q3. Let A, B and $C \subseteq B$ be sets. We make B^A into a probability space by selecting functions from A to B uniformly at random.
(a) What is the probability that a random f is an injection?
(b) What is the probability that $f(A) \subseteq C$ for a random f?
Express answers in terms of $a = |A|$, $b = |B|$ and $c = |C|$.

Q4. A permutation is given in cycle form. Write it in two line form and find its tenth power. Three versions were given depending on ID number.

$$(1,3,7)(2,9,4,8)(5,6) \qquad (1,2,5,4)(3,9)(6,8,7) \qquad (1,7)(2,4,9,6)(3,8,5).$$

Q5. Suppose X and Y are random variables with mean 0 and variance σ^2. Suppose that $\mathrm{Cov}(X,Y) = c$. Express the following in terms of σ and c.

$$\mathrm{E}(X^2) \qquad \mathrm{Var}(X+Y) \qquad \mathrm{Cov}(X+Y,\ X-Y)$$

Q6. A tree was drawn on the blackboard and the following were requested:
breadth first vertex sequence (BFV),
depth first vertex sequence (DFV),
preorder sequence of vertices (PREV),
the ranks of the leaves.
The root of the tree was F. From F, edges led to H, C and G. From H, edges led to D and A. From C, an edge led to E. From B, an edge led to B.

Q7. The permutations of $\{1,2,3,4,5,6\}$ are listed in lexicographic order. What is the rank of the following permutation? [Choice depends on ID number.]

$$2,1,5,3,6,4 \qquad 3,1,2,6,4,5 \qquad 4,2,1,3,6,5 \qquad 1,6,4,2,3,5.$$

Q8. We have a fair coin and a coin that is biased 2/3 heads and 1/3 tails. The following procedure is carried out.

(1) Choose a coin at random.
(2) Toss the coin. If the result is tails, accept the result of the toss and stop.
(3) Otherwise, toss the coin again and accept the result of the toss.

Answer the following.

(a) Draw the decision tree, labeling the edges with probabilities and the vertices with coin type or toss result, as appropriate.
(b) What is the probability of accepting heads?
(c) What is the probability that the biased coin was chosen, given that we accepted heads?

Q9. Give **SIMPLE** graphs satisfying the conditions for each problem **OR** explain why none exist.

(a) A graph with 3 vertices and 1 edge.
(b) A graph with 3 vertices and 4 edges.
(c) A graph with 3 vertices and 2 connected components.
(d) A graph G with 4 vertices and one H with 5 vertices such that G is a subgraph of H.
(e) A graph G with 4 vertices and one H with 5 vertices such that G is isomorphic to H.

Q10. Draw the graph shown here on your paper. The labels on the edges are costs. I haven't bothered to label vertices except for v_0 Recall that Prim's algorithm starts with v_0 and grows a minimum cost spanning tree.

(a) List the costs of the edges in the order Prim's algorithm adds them to the tree.

(b) Highlight or shade the edgs of the minimum cost spanning tree.