

- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK, but calculators ARE allowed.
- **You must show your work to receive credit.**

1. (36 pts) A *five digit number* is a sequence of five digits, the first of which is **NOT** zero. Thus, 12345 and 10101 are valid but 01234 and 1234 are **NOT** valid.

- (a) How many five digit numbers are there?
- (b) How many five digit numbers have all digits different? (as in 12345 but not 10101)
- (c) How many five digit numbers have no digit appearing just once? (So 11111 and 10010 are okay, but 10111 and 12312 are not.)

2. (16 pts.) Nine people, including Alice, are to be divided into two teams of four people each, plus a referee. If all divisions are equally likely, what is the probability that Alice is the referee? (No, it doesn't matter if the teams are distinguishable or not.)

Be sure to explain how you got your answer.

3. (24 pts.) An integer k from 1 to 9 is picked uniformly at random.

Let $X(k) = 1$ if k is odd and $X(k) = 0$ if k is even.

Let $Y(k)$ be the remainder when k is divided by 3.

- (a) Draw a table like the one here and fill in the probabilities.

$X \setminus Y$	0	1	2
0	?	?	?
1	?	?	?

- (b) Compute $\text{Cov}(X, Y)$.

4. (24 pts.) Suppose a strictly decreasing function $f : \{1, 2\} \rightarrow \{1, 2, \dots, n\}$ is chosen uniformly at random. The random variable X is defined by $X(f) = f(1)$.

- (a) Describe choosing f in terms of choosing subsets of a set. (Specify the set, what subsets are chosen, and how they are chosen.) If S is a subset associated with f , what is X in terms of S ?

- (b) Derive the formula

$$P(X = k) = \begin{cases} (k-1) / \binom{n}{2} & \text{for } 1 \leq k \leq n, \\ 0 & \text{for } k < 1 \text{ and } k > n. \end{cases}$$

END OF EXAM