

1. It is the amount of money, in cents, that the street vendor takes in (or “collects”, or “earns”, etc.) that day. If you did not give the units (cents), you do not get full credit.
2. The vectors  $\langle -1, 2, 0 \rangle$  and  $\langle 0, 0, 1 \rangle$  are parallel to the plane. Therefore a normal to the plane is given by

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 2\mathbf{i} + \mathbf{j}.$$

Hence an equation for the plane is

$$0 = \langle 2, 1, 0 \rangle \cdot \langle x - 1, y - 0, z - 0 \rangle = 2x + y - 2.$$

Thus the answer is  $2x + y = 2$ . Of course, this could be multiplied by a constant; for example,  $6x + 3y = 6$  is acceptable.

3. Calling the vectors  $\mathbf{u}$  and  $\mathbf{v}$  and the angle  $\theta$ ,

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|} = \frac{3 - 4 + 3}{(\sqrt{1^2 + 2^2 + 3^2})^2} = \frac{2}{14}.$$

Thus  $\theta = \cos^{-1}(1/7)$ .

4. Writing  $F(x, y) = x^3 - x^2y + y^3$ , we have  $dy/dx = -F_x/F_y$ . Since  $F_x = 3x^2 - 2xy$  and  $F_y = -x^2 + 3y^2$ , we have  $F_x(2, 1) = 8$  and  $F_y = -1$ . Hence the answer is  $dy/dx = 8$ .

5. (a)  $\{(x, y, z) \mid x + 2y + z > 0\}$ .

(b) Since  $\nabla g(x, y, z) = \left\langle \frac{1}{x+2y+z}, \frac{2}{x+2y+z}, \frac{1}{x+2y+z} \right\rangle$ , we have  $\nabla g(2, 1, -3) = \langle 1, 2, 1 \rangle$ .

(c) Since a vector in the given direction is  $\langle -2, -1, 3 \rangle$  and its length is  $\sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$ , we have  $\mathbf{u} = (1/\sqrt{14})\langle -2, -1, 3 \rangle$ .

Thus  $D_{\mathbf{u}}g(2, 1, 3) = (1/\sqrt{14})(-2 - 2 + 3) = -1/\sqrt{14}$ .

6.  $\partial z/\partial s = (\partial z/\partial x)(\partial x/\partial s) + (\partial z/\partial y)(\partial y/\partial s)$ . At  $(s, t) = (1, 2)$  we have  $(x, y) = (3, 2)$ . Computing the various partials:

$$\frac{\partial z}{\partial x} = 2x + y = 8 \quad \frac{\partial z}{\partial y} = x + 2y = 7 \quad \frac{\partial x}{\partial s} = 1 \quad \frac{\partial y}{\partial s} = t = 2.$$

Thus the answer is  $8 * 1 + 7 * 2 = 22$ .

7. With  $f(x, y) = 4x^2 - y^2 + 2y$ , we have  $f_x(-1, 2) = -8$  and  $f_y(-1, 2) = -4 + 2 = -2$ . Thus the equation is  $z - 4 = -8(x + 1) - 2(y - 2)$ . If you wish, you can rewrite this as  $z = -8x - 2y$  or  $8x + 2y + z = 0$ .

8. We have  $f_x = 3(x-1)(x+1)y$  and  $f_y = x^3 - 3x + 2y$ . From  $f_x = 0$  we have  $x = 1$  or  $x = -1$  or  $y = 0$ . We use each of them in  $f_y = 0$ :
- If  $x = 1$ , then  $f_y = -2 + 2y = 0$  gives  $y = 1$ .
  - If  $x = -1$ , then  $f_y = 2 + 2y = 0$  gives  $y = -1$ .
  - If  $y = 0$ , then  $f_y = x^3 - x = 0$  gives  $x = 0$  or  $x = \sqrt{3}$  or  $x = -\sqrt{3}$ .

Thus the five points are

$$(1, 1) \quad (-1, -1) \quad (0, 0) \quad (\sqrt{3}, 0) \quad (-\sqrt{3}, 0).$$

9. Using Lagrange multipliers we have the equation  $\langle 8x^3, 2y \rangle = \lambda \langle 2x, 2y \rangle$ , which can be written as

$$4x^3 = \lambda x \quad \text{and} \quad y = \lambda y.$$

The first of these tells us that  $x = 0$  or  $4x^2 = \lambda$ . The second tells us that  $y = 0$  or  $\lambda = 1$ . This leads to four possibilities which must be combined with the constraint  $x^2 + y^2 = 1$ :

- $x = 0$ ,  $y = 0$  and  $x^2 + y^2 = 1$ : This is impossible.
- $x = 0$ ,  $\lambda = 1$  and  $x^2 + y^2 = 1$ : Since  $x = 0$ , it follows that  $y = \pm\sqrt{1-0^2} = \pm 1$  and so  $f = 1$ .
- $4x^2 = \lambda$ ,  $y = 0$  and  $x^2 + y^2 = 1$ : Since  $y = 0$ , it follows that  $x = \pm 1$  and so  $f = 2$ .
- $4x^2 = \lambda$ ,  $\lambda = 1$  and  $x^2 + y^2 = 1$ : It follows that  $4x^2 = 1$  and so  $x = \pm 1/2$ . Thus  $y = \pm\sqrt{1-(1/2)^2} = \pm\sqrt{3}/2$  and  $f = 2 \times (1/2)^4 + (\sqrt{3}/2)^2 = 1/8 + 3/4 = 7/8$ .

Looking at the values of  $f$  we see:

absolute minimum:  $f = 7/8$  at the four points  $(\pm 1/2, \pm\sqrt{3}/2)$

absolute maximum:  $f = 2$  at the two points  $(\pm 1, 0)$ .

(At the points  $(0, \pm 1)$  we have the *local* minimum  $f = 1$ , but you were not asked to identify those.)