1. (a) This can be done in various ways:
   - Best is to take a value on each side of the (50,10) entry and form the finite difference:
   $$w_s(50,10) \approx \frac{37-21}{60-40} = \frac{16}{20} \quad \text{and} \quad w_t(50,10) \approx \frac{36-19}{15-5} = \frac{17}{10}.$$
   - Not as good is to take a value at the point and a point on one side (but this is still acceptable):
   $$w_s(50,10) \approx \frac{37-29}{60-50} = \frac{8}{10} \quad \text{or} \quad w_s(50,10) \approx \frac{29-21}{50-40} = \frac{8}{10}$$
   and
   $$w_t(50,10) \approx \frac{36-29}{15-10} = \frac{7}{5} \quad \text{or} \quad w_t(50,10) \approx \frac{29-19}{10-5} = \frac{10}{5}.$$

   (b) The units of w_s are feet per knot and those of w_t feet per hour.
   If you have singular or plural where you should not (foot, knots, hours), you will still receive credit.

   (c) The answer is $w(50,10) + w_s(50,10) \times (-1) + w_t(50,15) \times 1$. You should plug in the numbers you got in (a).

2. By the chain rule
$$\begin{aligned} f'(1) &= g_x(x(1),y(1))x'(1) + g_y(x(1),y(1))y'(1) \\ &= 1 \times 1 + (-2) \times 3 = -5. \end{aligned}$$

   (b) Since $f_{xy} = f_{yx}$, we'll compute f_x first. It is $3x^2y$. Thus $f_{xy} = \partial(3x^2y)/\partial y = 3x^2$.

   (c) $|\langle 1,2\rangle| = \sqrt{1^2+2^2} = \sqrt{5}$. Thus $\mathbf{u} = \langle 1/\sqrt{5}, 2/\sqrt{5}\rangle$. Since $\nabla f = \langle 2x-2y, 2x\rangle$, we have $\nabla f(0,1) = \langle -2, 0\rangle$ Finally $D_{\mathbf{u}} f(0,1) = \nabla f(0,1) \cdot \mathbf{u} = -2/\sqrt{5}$.

3. The gradient is $\langle 6x, 2y, 4z\rangle$, which equals $\langle 6,-8,4\rangle$ at $(1,-4,1)$. Thus the equation of the plane is
$$0 = \langle 6,-8,4\rangle \cdot \langle x-1, y+4, z-1\rangle = 6x - 8y + 4z - 42,$$
which can be rewritten as $3x - 4y + 2z = 21$. Any of these forms, including the dot product form, is acceptable.

4. Since we are given the critical points, we only need to compute f_{xx}, f_{xy}, f_{yy} and $D = f_{xx}f_{yy} - (f_{xy})^2$ there.

quantity	at (x,y)	at $(0,0)$	at $(1,\sqrt{2})$	at $(1,-\sqrt{2})$
f_{xx}	-2	-2	-2	-2
f_{xy}	$-2y$	0	$-2\sqrt{2}$	$2\sqrt{2}$
f_{yy}	$2x-2$	-2	0	0
D	$4(1-x-y^2)$	4	-8	-8

By the second derivative test, $(0,0)$ is a local maximum and the other two are saddle points.