1. (a) This can be done in various ways:
   - Best is to take a value on each side of the (50,15) entry and form the finite difference:
   $$w_s(50,15) \approx \frac{47-25}{60-40} = \frac{22}{20} \quad \text{and} \quad w_t(50,15) \approx \frac{40-29}{20-10} = \frac{11}{10}.$$
   - Not as good is to take a value at the point and a point on one side (but this is still acceptable):
   $$w_s(50,15) \approx \frac{47-36}{60-50} = \frac{11}{10} \quad \text{or} \quad w_s(50,15) \approx \frac{36-25}{50-40} = \frac{11}{10}$$
   and
   $$w_t(50,15) \approx \frac{40-36}{20-15} = \frac{4}{5} \quad \text{or} \quad w_t(50,15) \approx \frac{36-29}{15-10} = \frac{7}{5}.$$

   (b) The units of w_s are feet per knot and those of w_t feet per hour.
   If you have singular or plural where you should not (foot, knots, hours), you will still receive credit.

   (c) The answer is $w(50,15) + w_s(50,15) \times 1 + w_t(50,15) \times (-1)$. You should plug in the numbers you got in (a).

2. By the chain rule
$$\begin{aligned} f'(1) &= g_x(x(1),y(1))x'(1) + g_y(x(1),y(1))y'(1) \\ &= 2 \times 2 + (-1) \times 1 = 3. \end{aligned}$$

   (b) Since $f_{xy} = f_{yx}$, we'll compute f_y first. It is $2xy$. Thus $f_{xy} = \partial(2xy)/\partial x = 2y$.

   (c) $|\langle 2,1\rangle| = \sqrt{2^2+1^2} = \sqrt{5}$. Thus $\mathbf{u} = \langle 2/\sqrt{5}, 1/\sqrt{5}\rangle$. Since $\nabla f = \langle 2x+2y, 2x\rangle$, we have $\nabla f(1,0) = \langle 2,2\rangle$ Finally $D_{\mathbf{u}}f(1,0) = \nabla f(1,0)\cdot\mathbf{u} = 6/\sqrt{5}$.

3. The gradient is $\langle 4x, 6y, 2z\rangle$, which equals $\langle 4,-6,8\rangle$ at $(1,-1,4)$. Thus the equation of the plane is
$$0 = \langle 4,-6,8\rangle \cdot \langle x-1, y+1, z-4\rangle = 4x - 6y + 8z - 42,$$
which can be rewritten as $2x - 3y + 4z = 21$. Any of these forms, including the dot product form, is acceptable.

4. Since we are given the critical points, we only need to compute f_{xx}, f_{xy}, f_{yy} and $D = f_{xx}f_{yy} - (f_{xy})^2$ there.

quantity	at (x,y)	at $(0,0)$	at $(\sqrt{2},1)$	at $(-\sqrt{2},1)$
f_{xx}	$2-2y$	2	0	0
f_{xy}	$-2x$	0	$-2\sqrt{2}$	$2\sqrt{2}$
f_{yy}	2	2	2	2
D	$4(1-y-x^2)$	4	-8	-8

By the second derivative test, $(0,0)$ is a local minimum and the other two are saddle points.