

1. (a) No: You cannot add a vector and a scalar ($\mathbf{b} \cdot \mathbf{c}$ is a scalar).
- (b) No: the cross product is only defined for vectors in \mathbb{R}^3 .
- (c) Yes: this is a vector in \mathbb{R}^3 .
- (d) No: \mathbf{a} is in \mathbb{R}^2 but $\mathbf{v} \times \mathbf{w}$ is in \mathbb{R}^3 , so we cannot take their dot product.

2. (a) The answer is $3\mathbf{a}/|\mathbf{a}| = (3/\sqrt{5})\mathbf{a} = \langle 6/\sqrt{5}, 3/\sqrt{5} \rangle$.

- (b) We need a vector $\mathbf{b} = \langle b_1, b_2 \rangle$ so that

$$0 = \mathbf{a} \cdot \mathbf{b} = 2b_1 + b_2.$$

Thus $b_2 = -2b_1$. Any nonzero b_1 with $\mathbf{b} = \langle b_1, -2b_1 \rangle$ is correct; for example, $\langle 1, -2 \rangle = \mathbf{i} - 2\mathbf{j}$.

3. Vectors for two sides of the triangle are $\mathbf{c} = \vec{AB} = \langle -1, 3, 0 \rangle$ and $\mathbf{b} = \vec{AC} = \langle 2, 0, 1 \rangle$. Since the area is $(|\mathbf{b}||\mathbf{c}|\sin\theta)/2$, we can proceed in one of two ways.

(i) $\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 1 \\ -1 & 3 & 0 \end{vmatrix} = \langle -3, -1, 6 \rangle$ and the area is $\sqrt{3^2 + 1^2 + 6^2}/2$. You can

leave the answer this way or simplify: $\sqrt{46}/2$.

- (ii) Since $|\mathbf{b}| = \sqrt{5}$, $|\mathbf{c}| = \sqrt{10}$ and $\mathbf{b} \cdot \mathbf{c} = -2$, it follows that $\cos\theta = -2/\sqrt{50}$. Thus, since $\sin^2\theta + \cos^2\theta = 1$, the area is

$$\frac{|\mathbf{b}||\mathbf{c}|\sin\theta}{2} = \frac{\sqrt{10}\sqrt{5}\sqrt{1-4/50}}{2} = \frac{\sqrt{46}}{2}.$$

Remark: You may have chosen different sides of the triangle, different directions on the sides, or different order in the cross product. Nevertheless, you should get the same area and, to within sign, the same cross or dot product.

4. The equation is $\langle 1, 1, 2 \rangle \cdot \langle x, y, z \rangle = \langle 1, 1, 2 \rangle \cdot \langle 2, -1, 1 \rangle$ and so we have $x + y + 2z = 3$.

5. We can use the formula $\frac{|ax_0 + by_0 + cz_0 + d|}{|\langle a, b, c \rangle|}$ to obtain

$$\frac{|2 * 1 + (-3) * 2 + 1 * 3 - 3|}{\sqrt{2^2 + (-3)^2 + 1^2}} = \frac{|-4|}{\sqrt{14}} = \frac{4}{\sqrt{14}}.$$