

1. (a) No: $\mathbf{a} \cdot \mathbf{w}$ makes no sense since $\mathbf{a} \in \mathbb{R}^3$ and $\mathbf{w} \in \mathbb{R}^2$.
 (b) Yes: this is a vector in \mathbb{R}^3 .
 (c) No: the cross product is only defined for vectors in \mathbb{R}^3 .
 (d) No: since $(\mathbf{a} \cdot \mathbf{b})$ is a scalar, we cannot take its cross product with \mathbf{c} .
2. (a) The answer is $2\mathbf{a}/|\mathbf{a}| = (2/\sqrt{5})\mathbf{a} = \langle 2/\sqrt{5}, 4/\sqrt{5} \rangle$.
 (b) We need a vector $\mathbf{b} = \langle b_1, b_2 \rangle$ so that

$$0 = \mathbf{a} \cdot \mathbf{b} = b_1 + 2b_2.$$

Thus $b_1 = -2b_2$. Any nonzero b_2 with $\mathbf{b} = \langle -2b_2, b_2 \rangle$ is correct; for example, $\langle -2, 1 \rangle = -2\mathbf{i} + \mathbf{j}$.

3. Vectors for two sides of the triangle are $\mathbf{c} = \vec{AB} = \langle 1, 2, 0 \rangle$ and $\mathbf{b} = \vec{AC} = \langle 1, 0, 3 \rangle$. Since the area is $(|\mathbf{b}| |\mathbf{c}| \sin \theta)/2$, we can proceed in one of two ways.

(i) $\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 3 \\ 1 & 2 & 0 \end{vmatrix} = \langle -6, 3, 2 \rangle$ and the area is $\sqrt{6^2 + 3^2 + 2^2}/2$. You can leave the answer this way or simplify: $\sqrt{49}/2 = 7/2$.

- (ii) Since $|\mathbf{b}| = \sqrt{10}$, $|\mathbf{c}| = \sqrt{5}$ and $\mathbf{b} \cdot \mathbf{c} = 1$, it follows that $\cos \theta = 1/\sqrt{50}$. Thus, since $\sin^2 \theta + \cos^2 \theta = 1$, the area is

$$\frac{|\mathbf{b}| |\mathbf{c}| \sin \theta}{2} = \frac{\sqrt{10} \sqrt{5} \sqrt{1 - 1/50}}{2} = \frac{\sqrt{49}}{2} = 7/2.$$

Remark: You may have chosen different sides of the triangle, different directions on the sides, or different order in the cross product. Nevertheless, you should get the same area and, to within sign, the same cross or dot product.

4. The equation is $\langle 2, -1, 1 \rangle \cdot \langle x, y, z \rangle = \langle 2, -1, 1 \rangle \cdot \langle 1, 2, 1 \rangle$ and so we have $2x - y + z = 1$.

5. We can use the formula $\frac{|ax_0 + by_0 + cz_0 + d|}{|\langle a, b, c \rangle|}$ to obtain

$$\frac{|3 * 1 + 2 * 2 + 1 * (-3) - 2|}{\sqrt{1^2 + 2^2 + (-3)^2}} = \frac{2}{\sqrt{14}}.$$