1. True: (a) (c) (d) (e) False: (b) (f)

2. If $t > 2$, $+1$ and $-1$ are zeroes of $x^2 - 1$ in $integers_t$. Thus we have at least the eight zeroes obtained by the eight possible sign choices in $(\pm 1, \pm 1, \pm 1)$.

3. Suppose $x, y \in A \cap B$ and $r \in R$. Then $x, y \in A$ and $x, y \in B$. Since $A$ is an ideal, $x - y \in A$ and also $rx, xr \in A$. Likewise for $B$. Hence $x - y \in A \cap B$ and also $rx, xr \in A \cap B$. Thus $A \cap B$ is an ideal.

   Variations are possible. For example, one could replace the $x - y$ statements with: Since the intersection of subgroups is a subgroup, $A \cap B$ is a subgroup under addition.

4. I'll use commutativity in $D$ without explicitly mentioning it.
   (a) We have $\varphi(ab) = a^2b^2 = \varphi(a)\varphi(b)$ and $\varphi(a + b) = a^2 + 2ab + b^2 = \varphi(a) + \varphi(b)$ since $2ab = 0$ because $D$ has characteristic 2.
   (b) Suppose $\varphi(a) = \varphi(b)$. Then $a^2 = b^2$ and so $(a - b)^2 = a^2 - 2ab - b^2 + 2b^2 = a^2 - b^2 = 0$ Since $D$ has no zero divisors and $(a - b)^2 = 0$, it follows that $a - b = 0$ and so $a = b$.

       Variations are possible. For example, $a^2 = b^2$ gives us $0 = a^2 - b^2 = (a + b)(a - b)$ and so the lack of zero divisors gives us $a = \pm b$. However, $-x = x - 2x = x$ and so $a = b$.
   (c) The degree of $\varphi(a)$ is always even, hence no polynomials of odd degree are in the image. Aside: In fact the image is precisely $\mathbb{Z}_2[x^2]$ because, as you should be able to prove), $\varphi(p(x)) = p(x)^2 = p(x^2)$.