1. True: (a) (c) (d) (e) False: (b) (f)

2. If \( t > 2 \), +1 and −1 are zeroes of \( x^2 - 1 \) in integers. Thus we have at least the eight zeroes obtained by the eight possible sign choices in \((±1, ±1, ±1)\).

3. Suppose \( x, y \in A \cap B \) and \( r \in R \). Then \( x, y \in A \) and \( x, y \in B \). Since \( A \) is an ideal, \( x - y \in A \) and also \( rx, xr \in A \). Likewise for \( B \). Hence \( x - y \in A \cap B \) and also \( rx, xr \in A \cap B \). Thus \( A \cap B \) is an ideal.

Variations are possible. For example, one could replace the \( x - y \) statements with:

Since the intersection of subgroups is a subgroup, \( A \cap B \) is a subgroup under addition.

4. I’ll use commutativity in \( D \) without explicitly mentioning it.

(a) We have \( \varphi(ab) = a^2b^2 = \varphi(a)\varphi(b) \) and \( \varphi(a + b) = a^2 + 2ab + b^2 = \varphi(a) + \varphi(b) \) since \( 2ab = 0 \) because \( D \) has characteristic 2.

(b) Suppose \( \varphi(a) = \varphi(b) \). Then \( a^2 = b^2 \) and so \( (a - b)^2 = a^2 - 2ab - b^2 + 2b^2 = a^2 - b^2 = 0 \) Since \( D \) has no zero divisors and \( (a - b)^2 = 0 \), it follows that \( a - b = 0 \) and so \( a = b \).

Variations are possible. For example, \( a^2 = b^2 \) gives us \( 0 = a^2 - b^2 = (a + b)(a - b) \) and so the lack of zero divisors gives us \( a = ±b \). However, \( -x = x - 2x = x \) and so \( a = b \).

(c) The degree of \( \varphi(a) \) is always even, hence no polynomials of odd degree are in the image. Aside: In fact the image is precisely \( \mathbb{Z}_2[x^2] \) because, as you should be able to prove), \( \varphi(p(x)) = p(x)^2 = p(x^2) \).