- Please put your name and ID number on your blue book.
- CLOSED BOOK, but BOTH SIDES of one page of notes are allowed.
- Calculators are NOT allowed.
- *In a multipart problem, you can do later parts without doing earlier ones.*
- **You must show your work to receive credit.**

1. (18 pts.) Which are TRUE and which are FALSE? Do NOT give reasons.
   (a) $x^5 + 9x^2 + 3$ is irreducible over $\mathbb{Z}[x]$.
   (b) For any ring R, $\langle a \rangle = aR$.
   (c) In an integral domain, every maximal ideal is prime.
   (d) Every finite integral domain is a field.
   (e) Every principal ideal domain is a unique factorization domain.
   (f) If $\varphi : R \to S$ is a ring homomorphism, then $\{x \mid \varphi(x) = 0\}$ is an ideal of S.

2. (8 pts.) Let m, n, k be integers greater than 2. Let 1 be the unity of $R = \mathbb{Z}_m \oplus \mathbb{Z}_n \oplus \mathbb{Z}_k$. Prove that $x^2 - 1$ has at least eight zeroes in R.
   Hint: $8 = 2^3$

3. (8 pts.) Let A and B be ideals of a ring R. Prove that the intersection $A \cap B$ is an ideal of R.

4. (16 pts.) Let D be an integral domain of characteristic 2.
   Define $\varphi : D \to D$ by $\varphi(r) = r^2$.
   Recall: A ring R has characteristic n means $n \cdot r = 0$ for all $r \in R$.
   (a) Prove that φ is a ring homomorphism.
   (b) Prove that φ is an injection; that is, $\varphi(a) = \varphi(b)$ implies $a = b$.
   It follows from (a) and (b) that φ is a ring automorphism when D is finite—you don't need to prove that
   (c) Prove that φ is *not* a ring automorphism when $D = \mathbb{Z}_2[x]$.

END OF EXAM