1. (a) FALSE. Existence of inverses; for example, $2^{-1}$.
   (b) FALSE. Must have same operation in group and subgroup.
   (c) TRUE.
   (d) FALSE. Odd times odd is even. Alternate: the identity is even.
   (e) FALSE. Give an example such as $(123)(45) \in S_5$, which has order 6.

2. (a) $\alpha = (152)(34)$
   (b) $|\alpha|$ is the least common of its cycle lengths when in disjoint form; that is, $\operatorname{lcm}(3, 2) = 6$.
   (c) It is odd. The parity is the same as the parity of the number of even cycles.

3.

| generator | 0 | 1 | 2 | 4 | 5 | 10 |
|---|---|---|---|---|---|---|
| order | 1 | 20 | 10 | 5 | 4 | 2 |

4. (a) Given two elements $X = axa^{-1}$ and $Y = aya^{-1}$ with $x, y \in H$, we have

$$XY^{-1} = axa^{-1}(aya^{-1})^{-1} = axa^{-1}(ay^{-1}a^{-1}) = a(xy^{-1})a^{-1}.$$

   This is in $aHa^{-1}$ since $xy^{-1} \in H$ because $H$ is a group.
   (b) $\varphi(x)\varphi(y) = axa^{-1}aya^{-1} = axya^{-1} = \varphi(xy)$.