1. (15 pts.) If $A$ is a subset of the complex numbers, let $A^*$ be the nonzero numbers in $A$. Recall that $\mathbb{Z}$ are the integers and $\mathbb{Q}$ are the rationals. Answer the following TRUE or FALSE. IF FALSE, YOU MUST GIVE A REASON TO RECEIVE CREDIT.

(a) $\mathbb{Z}^*$ with multiplication is a subgroup of $\mathbb{Q}^*$ with multiplication.
(b) $\mathbb{Q}^*$ with multiplication is a subgroup of $\mathbb{Q}$ with addition.
(c) The $2 \times 2$ nonsingular matrices over $\mathbb{Q}$ are a group under multiplication.
(d) The odd permutations in $S_9$ are a subgroup with the same operation as $S_9$.
(e) If $\alpha \in S_n$, then $|\alpha| \leq n$.

2. (12 pts.) Let $\alpha = (1534)(245)$ be an element of $S_5$.

(a) Write $\alpha$ as a product of disjoint cycles.
(b) Compute the order of $\alpha$; that is, compute $|\alpha|$.
   (This can be done without doing (a), but it is easier if you do (a).)
(c) Determine if $\alpha$ is even or odd and give a reason for your answer.

3. (11 pts.) For each subgroup of $\mathbb{Z}_{20}$, give its order and a generator.

4. (12 pts.) Let $G$ be a group, $a \in G$ and $H \leq G$ (i.e., $H$ is a subgroup of $G$). Define $aHa^{-1} = \{aha^{-1} | h \in H\}$.

(a) Prove that $aHa^{-1} \leq G$.
(b) Define $\varphi : H \to aHa^{-1}$ by $\varphi(x) = axa^{-1}$.
   Prove that $\varphi(xy) = \varphi(x)\varphi(y)$.

END OF EXAM