- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK except for one page of notes.
- Calculators are NOT allowed.
- *In a multipart problem, you can do later parts without doing earlier ones.*
- **You must show your work to receive credit.**

1. (15 pts.) If A is a subset of the complex numbers, let A^* be the nonzero numbers in A. Recall that $\mathbb{Z}$ are the integers and $\mathbb{Q}$ are the rationals. Answer the following TRUE or FALSE.
   IF FALSE,YOU MUST GIVE A REASON TO RECEIVE CREDIT.
   (a) $\mathbb{Z}^*$ with multiplication is a subgroup of $\mathbb{Q}^*$ with multiplication.
   (b) $\mathbb{Q}^*$ with multiplication is a subgroup of $\mathbb{Q}$ with addition.
   (c) The 2×2 nonsingular matrices over $\mathbb{Q}$ are a group under multiplication.
   (d) The odd permutations in S_9 are a subgroup with the same operation as S_9.
   (e) If $\alpha \in S_n$, then $|\alpha| \leq n$.

2. (12 pts.) Let $\alpha = (1534)(245)$ be an element of S_5.
   (a) Write α as a product of disjoint cycles.
   (b) Compute the order of α; that is, compute $|\alpha|$.
   (This can be done without doing (a), but it is easier if you do (a).)
   (c) Determine if α is even or odd and give a reason for your answer.

3. (11 pts.) For each subgroup of $\mathbb{Z}_{20}$, give its order and a generator.

4. (12 pts.) Let G be a group, $a \in G$ and $H \leq G$ (i.e., H is a subgroup of G). Define $aHa^{-1} = \{aha^{-1} \mid h \in H\}$.
   (a) Prove that $aHa^{-1} \leq G$.
   (b) Define $\varphi : H \to aHa^{-1}$ by $\varphi(x) = axa^{-1}$.
   Prove that $\varphi(xy) = \varphi(x)\varphi(y)$.

END OF EXAM