

1. (a) True (b) False (c) False (d) True (e) False.

2. (a) $\alpha = (152)(34)$
 (b) $|\alpha|$ is the least common of its cycle lengths when in disjoint form; that is, $\text{lcm}(3, 2) = 6$.

3. (a) Let $a, b \in G^n$. By the definition of G^n , there are $g, h \in G$ such that $a = g^n$ and $b = h^n$. Thus $ab = g^n(h^n)^{-1} = g^n(h^{-1})^n$. Since G is abelian, this equals $(gh^{-1})^n$, which is in G^n .
 (b) If a is a rotation $a^3 = e$. If b is a reflection $b^2 = e$ and so $b^3 = b$. Thus $(D_3)^3$ contains the identity and all reflections but no rotations. Since the product of two different reflections is a rotation, $(D_3)^3$ is not closed under multiplication and hence not a group.

4. (a) Clearly G is a subset of the complex numbers under multiplication, so all we need to do is show that it is a subgroup.
 One proof: Suppose $z = e^{ri}$ and $w = e^{si}$ are in G . Then $zw^{-1} = e^{(r-s)i}$ is also in G .
 An alternate proof: Suppose $z, w \in G$. Then $|zw^{-1}| = |z|/|w| = 1$ and so $zw \in G$.
 (b) There is exactly one such subgroup, namely $\langle e^{2\pi i/n} \rangle$. In other words, the n th roots of unity.