Math 103A | Midterm Exam (50 points) | 26 October 2005

- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK except for one page of notes.
- Calculators are NOT allowed.
- *In a multipart problem, you can do later parts without doing earlier ones.*
- **You must show your work to receive credit.**

1. (10 pts.) If S is a subset of the complex numbers, let S^* be the nonzero numbers in S. Recall that $\mathbb{Z}$ are the integers and $\mathbb{Q}$ are the rationals. Answer the following TRUE or FALSE.
   (a) $\mathbb{Z}$ with addition is a subgroup of $\mathbb{Q}$ with addition.
   (b) $\mathbb{Q}^*$ with multiplication is a subgroup of $\mathbb{Q}$ with addition.
   (c) $\mathbb{Z}^*$ with multiplication is a subgroup of $\mathbb{Q}^*$ with multiplication.
   (d) For all $n > 0$, the even permutations in S_n form a group.
   (e) For all $n > 0$, the odd permutations in S_n form a group.

2. (8 pts.) Let $\alpha = (1543)(235)$ be an element of S_5.
   (a) Write α as a product of disjoint cycles.
   (b) Compute the order of α; that is, compute $|\alpha|$.
   (This can be done without doing (a), but it is easier if you do (a).)

3. (16 pts.) Let $n > 0$ be an integer and let G be an abelian group. Define the set G^n by $G^n = \{g^n \mid g \in G\}$.
   (a) Prove that G^n is a subgroup of G.
   (b) Let D_3 be the dihedral group of rotations and reflections of an equilateral triangle. Show that $(D_3)^3$ is not a subgroup of D_3.
   Hint: What elements of D_3 are in $(D_3)^3$?

4. (16 pts.) Let G be the set of complex numbers of absolute value 1.
   Recall from 20B: (You may find this helpful.) Complex numbers of absolute value 1 in polar coordinates have $r = 1$ and arbitrary angle θ. They can be written as $e^{\theta i}$. Also $e^{2\pi i} = 1$ and $e^{ri}e^{si} = e^{(r+s)i}$.
   (a) Prove that G is a group under multiplication.
   (b) For all integers $n > 0$, find a subgroup of G of order n.

END OF EXAM