

# Graphs

Graph theory is one of the most widely applicable areas of mathematics. Its concepts and terminology are used in many areas to help formulate and clarify ideas. Graph theory theorems find application in a wide range of fields, particularly the newer scientific disciplines.

The notion of a “graph” is deceptively simple: It is a collection of points (called “vertices”) that are joined by lines (called “edges”). Often all that matters about the edges is which two vertices they join, not their length or how they curve. This concept is deceptive because it seems unlikely that such a simple, general notion could have an interesting theory or be of any use. Simplicity is important. Scientists often try to find the simplest workable solution both in formulating theories and designing experiments. Mathematicians try to find the simplest concepts that usefully encompass what they are studying. A computer scientist, being part scientist and part mathematician, also looks for simplicity. Why is there this push for simplicity?

1. Many scientists believe that the underlying laws of the universe should exhibit elegance and simplicity.
2. The more complicated a construction is, the more likely it is to malfunction. Some familiar examples of this are experimental apparatus, algorithms and computer programs.
3. A simple concept is usually more flexible than a complex one and so can be applied in more new situations. Common examples in which simplicity is a virtue are definitions, scientific theories and data structures.

“Simple” should not be confused with “unsophisticated.” Special relativity, complex variables and context-free grammars are all simple; but none of them are unsophisticated.

We’ll introduce some of the basic concepts in graph theory in Chapter 5 and then discuss some theory and applications in Chapter 6. To thoroughly discuss applications of graphs in computer science would require a very large book. Another would be needed to discuss the purely mathematical aspects of graph theory. We’ve picked a variety of important topics from different areas of graph theory and computer science. The topics we’ve chosen reflect our perceptions of what you should learn and also our own interests. These topics are

- **Spanning trees:** an important tool in combinatorial algorithms;
- **Graph coloring:** a subject with pretty results and a relatively long history;
- **Planarity:** a deep subject with connections to graph coloring;
- **Flows in networks:** an important application of graphs;
- **Random graphs:** the properties of typical graphs;
- **Finite state machines:** an important concept for formal languages and compiler design.