The sections refer to Stewart's calculus text.

Suppose we are given the differential equation $y' = F(x, y)$ with initial condition $y(x_0) = y_0$. Euler's method, discussed in Section 9.2, produces a sequence of approximations $y_1, y_2, \ldots$ to $y(x_1), y(x_2), \ldots$ where $x_n = x_0 + nh$ are equally spaced points.

This is almost the left endpoint approximation in numerical integration (Section 7.7). To see this, suppose that we have an approximation y_{n-1} for $y(x_{n-1})$, and that we want an approximation for $y(x_n)$. Integrate $y' = F(x, y)$ from x_{n-1} to x_n and use the left endpoint approximation:

$$y(x_n) - y(x_{n-1}) = \int_{x_{n-1}}^{x_n} F(x, y)\, dx \approx hF(x_{n-1}, y(x_{n-1})).$$

Now we have a problem that did not arise in numerical integration: We don't know $y(x_{n-1})$. What can we do? We replace $y(x_{n-1})$ with the approximation y_{n-1} to obtain

$$y(x_n) - y_{n-1} \approx hF(x_{n-1}, y_{n-1}).$$

Rearranging and calling the approximation to $y(x_n)$ thus obtained y_n we have Euler's method:

$$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1}). \tag{1}$$

We know that the left endpoint approximation is a poor way to estimate integrals and that the Trapezoidal Rule is better. Can we use it here? Adapting the argument that led to (1) for use with the Trapezoidal Rule gives us

$$y_n = y_{n-1} + \frac{h}{2}\Big(F(x_{n-1}, y_{n-1}) + F(x_n, y_n)\Big). \tag{2}$$

You should carry out the steps. Unfortunately, (2) can't be used: We need y_n on the right side in order to compute it on the left!

Here is a way around this problem: First, use (1) to estimate ("predict") the value of y_n and call this prediction y_n^*. Second, use y_n^* in place of y_n in the right side of (2) to obtain a better estimate, called the "correction". The formulas are

$$\begin{aligned} &\text{(predictor)} \qquad y_n^* = y_{n-1} + hF(x_{n-1}, y_{n-1}) \\ &\text{(corrector)} \qquad y_n = y_{n-1} + \frac{h}{2}\Big(F(x_{n-1}, y_{n-1}) + F(x_n, y_n^*)\Big). \end{aligned} \tag{3}$$

This is an example of a *predictor-corrector* method for differential equations. Here are results for Example 9.2.3, the differential equation $y' = x+y$ with initial condition $y(0) = 1$:

step size	$y(1)$ by (1)	$y(1)$ by (3)
0.50	2.500000	3.281250
0.20	2.976640	3.405416
0.10	3.187485	3.428162
0.05	3.306595	3.434382
0.02	3.383176	3.436207
0.01	3.409628	3.436474

The correct value is 3.436564, so (3) is much better than Euler's method for this problem.