

1. Let $\mathbf{a} = \vec{OA}$ and $\mathbf{b} = \vec{OB}$. The area of the triangle is $\frac{|\mathbf{a} \times \mathbf{b}|}{2} = \frac{|-3\mathbf{i} + \mathbf{j} - \mathbf{k}|}{2} = \frac{\sqrt{11}}{2}$ and the cosine of the angle is $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{-3}{\sqrt{20}} = \frac{-3\sqrt{5}}{10}$.

Note: If you said the sine of the angle is $\frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}| |\mathbf{b}|}$, this would be only partially correct since this doesn't distinguish between an angle and its supplement.

2. You can do this in various ways. One way is to use the line to find two points in the plane other than the given point, e.g. $(0, 0, 0)$ and $(6, 3, 2)$, and then use the 3 points to determine the plane. Another would be to find two vectors parallel to the plane and use them to find a normal. At any rate, a normal to the plane is $\langle 3, -8, 3 \rangle$. Since the plane contains the origin, its equation is $3x - 8y + 3z = 0$.

3. The unit tangent vector is $\frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{\langle 2, 2, 1 \rangle}{3}$.

4. $z_u = f_x x_u + f_y y_u = f_x + 2uv f_y$
 $z_{uv} = \partial f_x / \partial v + 2u f_y + 2uv \partial f_y / \partial v = f_{xx} + 2uv f_{xy} + 2u f_y + 2uv (f_{xy} + 2uv f_{yy})$,
 which can be simplified slightly, if you wish, to
 $z_{uv} = f_{xx} + 2u f_y + 4uv f_{xy} + 4u^2 v^2 f_{yy}$.

5. (a) $\langle 4/5, -3/5 \rangle$ (b) $\langle 3/5, 4/5 \rangle$ and $\langle -3/4, -4/5 \rangle$

6. (a) We have $\langle 4/5, -3/5, 0 \rangle$.

- (b) We have an infinite number of solutions $\mathbf{u} = \mathbf{v}/|\mathbf{v}|$ where $\mathbf{v} = \langle 3, 4, t \rangle$ or $\mathbf{v} = \langle -3, -4, t \rangle$ and t is any real number.

7. A normal to the plane $z = f(x, y)$ is $\langle f_x, f_y, -1 \rangle = \langle 4, -3, -1 \rangle$ and so the plane is given by

$$0 = \langle 4, -3, -1 \rangle \cdot \langle x - 2, y - 1, z - 3 \rangle = 4x - 3y - z - 2.$$

8. $\int_0^{2\pi} \int_0^2 r^3 \cos^2 \theta \, dr \, d\theta$.

9. $\int_0^1 \int_0^1 f(x, y) \, dx \, dy + \int_1^2 \int_{y-1}^1 f(x, y) \, dx \, dy$.

10. $\int_0^e \int_0^{1/y} y e^{xy} \, dx \, dy = \int_0^e e^{xy} \Big|_{x=0}^{x=1/y} = \int_0^e (e - 1) \, dy = (e - 1)e$.

11. Solving the equations for x and y , we have $x = \frac{u+v}{2}$ and $y = \frac{v-u}{2}$. The Jacobian is $x_u y_v - x_v y_u = 1/2$. The domain is $\{(u, v) \mid 0 \leq u \leq 1, -1 \leq v \leq 1\}$. Thus the integral becomes $\int_{-1}^1 \int_0^1 \frac{u}{2(v+2)} \, du \, dv$.