- Print Name and ID number on your blue book.
- BOOKS and CALCULATORS are NOT allowed. Two pages of NOTES (both sides) are allowed.
- **You must show your work to receive credit.**

1. (10 pts.) The vertices of a triangle are $O(0,0,0)$, $A(0,1,1)$ and $B(1,0,-3)$.
   (a) Compute the area of the triangle.
   (b) Compute the size of angle AOB. You may leave trig functions in your answer.

2. (8 pts.) Find the equation of the plane passing through $(1,0,-1)$ and containing the line with symmetric equations $x = 2y = 3z$.

3. (8 pts.) Find the unit tangent vector to $\mathbf{r}(t) = 4\sqrt{t}\,\mathbf{i} + t^2\mathbf{j} + t\,\mathbf{k}$ at $t = 1$.

4. (8 pts.) Your are given that $z = f(x,y)$, $x = u + v$ and $y = u^2 v$. Use the chain rule to compute the following.

   $$\text{(a)}\quad z_u = \partial z/\partial u \qquad \text{(b)}\quad z_{uv} = \partial^2 z/\partial u \partial v.$$

   Of course, you will have to leave expressions such as f_x, f_{xy} and so on in your answer since the partial derivatives of f are not known.

5. (6 pts.) The function $f(x,y)$ satisfies $\nabla f(1,4) = \langle 4, -3\rangle$.
   (a) Find *all* unit vectors $\mathbf{u}$ such that $D_{\mathbf{u}} f(1,4)$ is a large as possible.
   (b) Find *all* unit vectors $\mathbf{u}$ such that $D_{\mathbf{u}} f(1,4) = 0$.

6. (6 pts.) The function $g(x,y,z)$ satisfies $\nabla g(1,4,3) = \langle 4, -3, 0\rangle$.
   (a) Find *all* unit vectors $\mathbf{u}$ such that $D_{\mathbf{u}} g(1,4,3)$ is a large as possible.
   (b) Find *all* unit vectors $\mathbf{u}$ such that $D_{\mathbf{u}} g(1,4,3) = 0$.

7. (8 pts.) Find the tangent plane to the surface $z = x^2 - y^3$ at $(2, 1, 3)$.

8. (8 pts.) Write $\iint_D x^2 \, dA$ as an iterated integral in polar coordinates where D is the interior of the circle of radius 2 centered at the origin.
Do NOT evaluate the integral—just change to polar coordinates.

9. (10 pts.) Change the order of integration in $\int_0^1 \int_0^{x+1} f(x, y) \, dy \, dx$.

10. (8 pts.) Calculate $\iint_D ye^{xy} dA$ where $D = \{(x, y) \mid 0 \leq xy \leq 1, \ 1 \leq y \leq e\}$.

11. (10 pts.) Use the change of variables $u = x - y$, $v = x + y$ to rewrite the following as an iterated integral over u and v.

$$\iint_D \frac{x - y}{x + y + 2} \, dA \quad \text{where} \quad D = \{(x, y) \mid 0 \leq x - y \leq 1, \ |x + y| \leq 1\}.$$

Do NOT evaluate the integral—just change variables.

END OF EXAM