1. Since the directional derivative is a dot product and $\cos 60° = 1/2$, the answer is $3/2$.

2. We want to evaluate f at $x(0,0) = 1$ and $y(0,0) = 0$. By the chain rule, $g_s = f_x x_s + f_y y_s = 1 \times 1 + 3 \times 2 = 7$.

3. $\nabla f = \vec{0}$ gives us $3x^2 - 3y = 0$ and $3y^2 - 3x = 0$. Thus $x^2 = y$ and $y^2 = x$. Squaring the first and substituting in the second, $x^4 = x$. Since $x^4 - x = x(x-1)(x^2+x+1)$, we have two values $x = 0$ and $x = 1$. Using $x^2 = y$, we get $y = 0$ and $y = 1$, respectively. Thus the critical values are $\langle 0, 0\rangle$ and $\langle 1, 1\rangle$.

4. (a) With $x^2 + y^2 + z^2 = g$, $\nabla f + \lambda \nabla g = \langle 1 + 2\lambda x, 3 + 2\lambda y, 2 + 2\lambda z\rangle$. Thus $x = -1/2\lambda$, $y = -3/2\lambda$ and $z = -2/2\lambda$ and so $g(x, y, z) = 14/4\lambda^2$. It follows that $4\lambda^2 = 1$ and so $2\lambda = \pm 1$. Hence the critical points are $\langle x, y, z\rangle$ is either $\langle 1, 3, 2\rangle$ or $\langle -1, -3, -2\rangle$. (The function values are $+14$ and -14, respectively, but you were not asked for them.)

   (b) The process is similar to (a) except the roles of $x^2 + y^2 + z^2$ and $x + 3y + 2z$ are reversed. In this case, $\langle x, y, z\rangle = \langle 1, 3, 2\rangle$

   (c) The constraint is a plane and f is the square of the distance to the origin. Hence we are finding the point on the plane closest to the origin.

5. By the chain rule or Math 20A, $df/ds = (df/dx)(dx/ds)$. By the formula for implicit differentiation, $dx/ds = -G_s/G_x$ and so $df/ds = \dfrac{-(df/dx)(\partial G/\partial s)}{\partial G/\partial x}$.