Math 20C Second Exam 50 points February 22, 2008

- Print Name and ID number on your blue book.
- BOOKS and CALCULATORS are NOT allowed. One side of one page of NOTES is allowed.
- **You must show your work to receive credit.**
- **Carry out numerical calculations fully.**

1. (6 pts.) Suppose $|\nabla f(\vec{x}_0)| = 3$ and the angle between the unit vector $\vec{u}$ and $\nabla f(\vec{x_0})$ is $60°$. Compute $D_{\vec{u}}\vec{f}$ at $\vec{x}_0$.

2. (8 pts.) There are functions $f(x, y)$, $x(s, t)$ and $y(s, t)$. Let $g(s, t) = f(x(s, t), y(s, t))$. Compute $\partial g/\partial s$ at $s = t = 0$ given the following values

$$\begin{array}{lll} f(0,0) = 1 & f_x(0,0) = 2 & f_y(0,0) = 3 \\ f(0,1) = 2 & f_x(0,1) = 3 & f_y(0,1) = 1 \\ f(1,0) = 0 & f_x(1,0) = 1 & f_y(1,0) = 3 \\ x(0,0) = 1 & x_s(0,0) = 1 & x_t(0,0) = 0 \\ y(0,0) = 0 & y_s(0,0) = 2 & y_t(0,0) = 3 \end{array}$$

3. (12 pts.) Find and classify the critical points of $f(x, y) = x^3 + y^3 - 3xy$.

4. (a) (10 pts.) Find the critical points of $f(x, y, z) = x + 3y + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 14$.

   (b) (6 pts.) Find the critical point of $f(x, y, z) = x^2+y^2+z^2$ subject to the constraint $x + 3y + 2z = 14$. (There is only one.)

   (c) (2 pts.) Interpret (b) in terms of planes and distances.

5. (6 pts.) Suppose you are given $f(x)$ and $G(x, s)$. The equation $G(x(s), s) = 0$ is used to determine x as a function of s. Thus we can think of f as a function of s, namely $f(x(s))$. Derive a formula for df/ds in terms of df/dx, $\partial G/\partial x$ and $\partial G/\partial s$.

END OF EXAM