For your information:

- 2^X denotes the power set of X; that is, the set of all subsets of X. It is also denoted by $\mathcal{P}(S)$.
- $\mathbb{Z}$ denotes the set of integers: 0, ± 1, $\pm 2, \ldots$.

1. Which of the following are true and which are false?
   (a) $\emptyset \subseteq \emptyset$ **TRUE**: The empty set is a subset of *every* set.
   (b) $\emptyset \in \emptyset$ **FALSE**: The empty set has *no* members.
   (c) $\emptyset \in 2^\emptyset$ **TRUE**: The empty set is a subset of *every* set and hence is in the power set of every set.
   (d) $\emptyset \subseteq 2^\emptyset$ **TRUE**: The empty set is a subset of *every* set.
   (e) $\mathbb{Z} \subseteq \mathbb{Z}$ **TRUE**: *Every* set is a subset of itself.
   (f) $\mathbb{Z} \in \mathbb{Z}$ **FALSE**: The members of $\mathbb{Z}$ are integers, not sets of integers.
   (g) $\emptyset \in \mathbb{Z}$ **FALSE**: The members of $\mathbb{Z}$ are integers, not sets of integers.
   (h) $\emptyset \subseteq \mathbb{Z}$ **TRUE**: The empty set is a subset of *every* set.

2. Which of the following are true statements about our world?
   - For "A if and only if B" to be true, we must have A and B both true or both false at the same time.
   - "If A, then B" is true if and only if B is true whenever A is true.
   - "A only if B" means "if A, then B". This is a confusing construction and should be avoided.

   (a) Dogs can fly if and only if pigs have wings.
   TRUE: Both sides are always false.
   (b) If dogs can fly, then pigs have wings.
   TRUE: We need to know that whenever dogs can fly, pigs have wings. Since dogs can never fly, this is true.
   (c) Dogs can fly only if pigs do not have wings.
   TRUE: Again, this is true since dogs can never fly.

3. Describe the following sets in English as simply as possible.
   (a) $2^\emptyset$ This is the set containing one element, and that element is the empty set.
   (b) $\{2k : k \in \mathbb{Z}\}$ This is the set of all even integers.
   (c) $\{k + 37 : k \in \mathbb{Z}\}$ This is the set of all integers.
   (d) $\{2k : k \in \mathbb{Z}\} \cap \{3k : k \in \mathbb{Z}\}$ This is the set of integers that are multiples of both 2 and 3; in other words, those which are multiples of 6.
   (e) $2\mathbb{Z} \cap 3\mathbb{Z}$ This is just (d) written in another notation.