1. (a) FALSE. $D[x]/\langle x\rangle \approx D$. Let $D=\mathbb{Z}$.
   (b) TRUE. See p.329.
   (c) TRUE. See p.294.
   (d) TRUE. See p.250.
   (e) FALSE. The square of $x+1+\langle x^2+1\rangle$ is zero.
   (f) FALSE. The converse is true.

2. Let $A \neq \{0\}$ be an ideal of F. Suppose $r \neq 0$ and $r \in A$. Then $s=(sr^{-1})r \in A$ for all $s \in F$ and so $A=F$.

3. (a) $\phi(r)+\phi(s)=6r+6s=6(r+s)=\phi(r+s)$ and $\phi(r)\phi(s)=6r6s=36rs=6rs=\phi(rs)$, where we have used the fact that $36=6$ in $\mathbb{Z}_{10}$.
   (b) In $\mathbb{Z}_{10}$, $6r=0$ if and only if it is a multiple of 10. This happens for $r=0$ and $r=5$. Thus $\text{Ker}\phi=\{0,5\}$.
   (c) The image of ϕ is $S=\{0,2,4,6,8\} \subset \mathbb{Z}_{10}$, not all $\mathbb{Z}_{10}$. In fact, $6=\phi(1)$ is the unity of the ring S.

4. Suppose $r^3=r$ and $s^3=s$.
   - Note that since the characteristic is 3, $3x=0$ for all $x \in R$.
     We have $(r-s)^3=r^3-3r^2s+3rs^2-s^3=r-s$.
   - $(rs)^3=r^3s^3=rs$.

   Note: If the two occurrences of "3" in the statement of the problem are replaced by a prime p, S is still a subring. When $p=2$, the fact that $x=-x+2x=-x$ is also needed.

5. Suppose $s \in \langle e\rangle$. Then $s=er$ for some $r \in R$. Thus $es=e^2r=er=s$.
   A somewhat subtle point: we need to know that $e \in \langle e\rangle$. (Since $\langle a\rangle=aR$, this may not be true for general a if R does not not have a unity.) In this case, $e=e^2=ee \in eR=\langle e\rangle$. Since the book only defined principal ideals for commutative rings with unity, you will NOT lose points if you did not prove $e \in \langle e\rangle$.