Math 20C - Calculus and Analytic Geometry - Spring 2021

Goal: Develop calculus in 2D & 3D

Some Class Details:

- Course links are on the CANVAS page.

Links:

1. Course website:
   (syllabus: contact info/schedule)

2. Discord Site:
   (A good place to ask questions)

3. Gradescope Site:
   (where quizzes will be released)

4. Zoom Links:
   (class, discussion, S1, office hours...)

Note: Press record
#5. WEBASSIGN:

(Where the HW happens.)

YOUR GRADE:

50% HOMEWORK SCORE.
50% QUIZ SCORE.

HOMEWORK SCORE:

= AVG OF TOP 8 HW 70s
(aka: drop the lowest)

QUIZ SCORE:

= the higher of

#1. AVG QUIZ %
OR #2. \( \frac{2}{3} \) (AVG of TOP 4 QUIZ %) + \( \frac{1}{3} \) (FINAL %).

QUESTIONS?
VECTORS in 2D & 3D

2D.

\[ \text{"y-coord."} = b \]
\[ (a,b) = P. \]
\[ a = \text{"x-coordinate"}. \]

For any point \((a,b)\) we make the vector from \((0,0)\) to \((a,b)\).
COMMON CONFUSION.

What is the difference between points and vectors?

A point: A spot in 2D/3D.
A vector: A magnitude & direction.
EVEN WORSE.

We will use the notation $(1,2,3)$ to refer to both

AND

this point

AND

this vector.

It you find that confusing or dishonest THAT IS OK.
SOME NOTATION.

The set of real numbers is denoted by \( \mathbb{R} \).

The set of pairs of real numbers is denoted by \( \mathbb{R}^2 \).

Triples of real numbers (respectively \( n \)-tuples) are denoted by \( \mathbb{R}^3 \) (resp. \( \mathbb{R}^n \)).

VECTOR ADDITION.

GEOMETRICALLY IN 2D.

(SIMILARLY IN 3D)
THE FORMULA (IN 3D).

\vec{v} = (-1, 2, \pi)
\vec{w} = (2, 1, 1)
\vec{v} + \vec{w} = (1, 4, \pi + 1)

(ADD EACH COORDINATE).

COMMENT. You can't add 2D & 3D vectors (or scalar + vector).

MULTIPLYING VECTORS.

METHOD #1 SCALAR MULTIPLICATION.

Let \( \vec{v} \in \mathbb{R}^2 \), AND let \( \lambda \in \mathbb{R} \).

(NOTATION \( \in \) means "in", read it as "let \( \vec{v} \) be an element in \( \mathbb{R}^2 \)".)
Scalar multiplication given a vector

\[ \lambda \vec{v} \in \mathbb{R}^2 \]

**FORMULA.** If \( \vec{v} = (a, b) \) then

\[ \lambda \vec{v} = (\lambda a, \lambda b) \in \mathbb{R}^2 \]

(Multiply each coord. by \( \lambda \))

**GEOMETRICALLY.**

(A) If \( \lambda > 0 \).

\( \lambda \) times longer in same direction.
(B) If \( \lambda \leq 0 \).

A times larger in opposite direction.

(Similar in 3D).

Lines in 2D & 3D.

Need
1. A point
2. A direction

gives a line.
Every other point on $L$ has the form $\overline{w} + t\overline{v}$ for some $t \in \mathbb{R}$.

(why??)
We can parametrize $L$ by the function:

$$f(t) = \overline{w} + t\overline{v}.$$  

(Works for any line through a point $\overline{w}$ with direction $\overline{v}$)

In our example:

$$f(t) = (2,1) + t(2, -1).$$  

$$= (2,1) + (2t, -t).$$  

$$= (2+2t, 1-t).$$

This line has $x$-intercept when $1-t=0$ so $t=1$.

$$f(1) = (4,0)$$  

$x$-intercept.