HOMEWORK 5 - MATH 100A - DUE MONDAY NOVEMBER 13TH

Problem 1. Let $H \leq G$ be a subgroup of G. Assume that the number of left cosets of H in G is 2 (i.e. $[G : H] = 2$, i.e. the *index of H in G* is 2). Prove that H is a normal subgroup of G.

Problem 2. (Herstein §2.6 # 7) If G is a cyclic group and N is a subgroup of G, show that G/N is cyclic.

Problem 3. Let $\mathbb{R}$ be the group of real numbers with addition, and $\mathbb{R}^{>0}$ be the group of positive real numbers with multiplication. Prove $\mathbb{R} \cong \mathbb{R}^{>0}$.

Problem 4. Let G be an abelian group. Prove that every subgroup of G is normal.

Problem 5. Let G be an abelian group with operation $+$. Let n be a positive integer. Define a function
$$\text{mult}_n \colon G \to G$$
$$\text{by } x \mapsto \text{mult}_n(x) = nx = (x + \cdots + x).$$
Prove that mult_n is a homomorphism, and describe the kernel of mult_n.

Problem 6. (From Herstein §3.2 #1) Show that if σ, τ are two disjoint cycles, then $\sigma\tau = \tau\sigma$.

Problem 7. (From Herstein §3.3 #2) If σ is a k-cycle, show that σ is an odd permutation if k is even, and is an even permutation if k is odd.

Problem 8. Find all the conjugacy classes in S_6, and find a representative for each conjugacy class.

Problem 9. (From Herstein §3.3 #8) Find a normal subgroup in A_4 of order 4.

Problem 10. In every group G the trivial subgroups $\{e\}, G \leq G$ are always normal. Define G to be *simple* if it has no nontrivial normal subgroups. Assume that G is a finite, simple, and **abelian** group. Prove that $|G| = p$, where p is prime.