Friday, December 11

{press record}

Our last class.

Remarks about the final.

- Practice Final (available on course website)
- I will have 3 office hours next week.
  - Wednesday: 10 am - 1 pm
- The final is cumulative and counts as 2 quiz scores. (Remember: you can drop your lowest 3 quiz scores)
Important themes from class:

**Week 1:** Vectors, dot product, length, projection.

**Week 2:** Matrices, determinants, the cross product.

**Week 3:** Differentiation, linear approximation.

**Week 4:** More differentiation, the chain rule.

**Week 5:** Gradients, directional derivatives, 2nd partial derivatives.

**Week 6:** Critical points, finding local extrema, 2nd derivative test, global mins/maxes.
Week 7: Lagrange multipliers!

Week 8: Acceleration & Arc length.

Week 9: Double integrals, Cavalieri’s principle, iterated integrals,

Week 10: Changing the order of integration & triple integrals (today!)

- The final will have 6-8 questions.
- Takes place on Friday 8-11 am

Questions?
§5.5 The triple integral

\[ R = [a,b] \times [c,d] \times [e,f]. \]

\[
\begin{align*}
\text{so} & \quad a \leq x \leq b \\
c & \leq y \leq d \\
e & \leq z \leq f
\end{align*}
\]

\[ C = \{ [a,b], [c,d], [e,f] \} \text{ in to } m \times n \times p \text{ equal pieces:} \]

\[
\begin{align*}
a &= x_0 \leq \cdots \leq x_i \leq \cdots \leq x_n = b & (\Delta x \text{ apart}) \\
c &= y_0 \leq \cdots \leq y_j \leq \cdots \leq y_u = d & (\Delta y \text{ apart}) \\
e &= z_0 \leq \cdots \leq z_k \leq \cdots \leq z_v = f & (\Delta z \text{ apart})
\end{align*}
\]
We can compute triple integrals by iterating:

$$\iiint_{R} f(x,y,z) \, dz \, dy \, dx \quad = \quad \lim_{n \to \infty} \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} f(x_i, y_j, z_k) \Delta x \Delta y \Delta z \right).$$

$$\iiint_{R} f(x,y,z) \, dz \, dy \, dx = \int_{a}^{b} \int_{c}^{d} \left( \int_{e}^{f} f(x,y,z) \, dz \right) \, dy \, dx.$$  

(when $R$ is a rectangle, can do this in any order.)
Example:

\[
\iiint_0^1 xy - yz + 2x \, dz \, dy \, dx
\]

\[
= \int_0^1 \int_0^2 \left( xy - yz + \frac{2x}{3} \right) \, dy \, dx
\]

\[
= \int_0^1 \int_0^2 \left( xy - yz + \frac{x}{2} \right) \, dy \, dx
\]

\[
= \int_0^1 \left. \left( \frac{xy^2}{2} - \frac{y^2}{2} + \frac{xy}{2} \right) \right|_0^2 \, dx
\]

\[
= \int_0^1 \frac{x \cdot 4}{2} - \frac{4}{2} + \frac{2x}{2} \, dx
\]

\[
= \int_0^1 2x - 1 + x \, dx = \left( x^2 - x + x^2 \right)_0^1
\]

\[
= 1 - 1 + \frac{1}{2} = \frac{1}{2}.
\]
There are many other regions in $\mathbb{R}^3$ that we can integrate over...

\[ \iiint_{\mathbf{R}} f(x,y,z) \, dz \, dy \, dx = \iint_{\mathbf{D}} \left( \int_{f(x,y)}^{f_2(x,y)} f(x,y,z) \, dz \right) \, dy \, dx \]
Remark: Integrating the function 1 over a region in 3D will compute its volume.

THANKS FOR THE GREAT QUARTER!