Math 31CH Spring 2018 Written Homework 1, due 4/13/2018 in HW box in the basement of AP&M by 4 pm

Reading

Sections 6.1-6.3 of the text.

Exercises to submit on Friday 4/13

Exercises from the text

Write out each answer as a careful proof, in full sentences.

Section 6.1: #2, 4
Section 6.2: #1(a)(c)(d), 2(a)(b), 3(c)(d)(e), 4, 12

Remarks: For 6.2 #1, exercise C below will be helpful in justifying that you have found exactly the points where the function has a local inverse.

Additional problems to hand in (not from the text)

A. Let X, Y, Z be sets and let $f : X \to Y$ be a function. Recall that an *inverse* to f is a function $g : Y \to X$ such that $g(f(x)) = x$ for all $x \in X$ and $f(g(y)) = y$ for all $y \in Y$. In this case we write f^{-1} for g. Recall that f is *onto* or *surjective* if for all $y \in Y$, there exists $x \in X$ such that $f(x) = y$. In addition, recall that f is *one-to-one* or *injective* if given $x_1, x_2 \in X$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

(a). Show that f has an inverse function if and only if f is both injective and surjective.

(b). Show that if f has an inverse function, the inverse function is unique.

B. Give a different proof of Chapter 6, Theorem 1.2 on page 245 of the text which does not rely on Proposition 1.1, but instead uses Exercise 6.1 #4.

C. Prove the converse of the inverse function theorem (Chapter 6, Theorem 2.1 on page 252 of the text). Namely, prove that (in the notation of the statement of that theorem) if the function $f : U \to \mathbb{R}^n$ is $\mathcal{C}^1$ and there is a neighborhood $V \subseteq U$ of $\vec{x}_0$ such that f has a $\mathcal{C}^1$ inverse function on V, then $Df(\vec{x}_0)$ is invertible.