MATH 31CH, **Exam 1** **April 21, 2015**

Name and PID: ____________________

Instructions:

- Write your Name and PID.
- **You may not use any electronic devices, textbooks, or notes.** If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped.
- You must justify your work (legibly) to receive credit.
- **You may NOT use homework problems (without proof) in your solutions.**

Good Luck!

Problem	Score	Out of
1		**10**
2		**10**
3		**5**
4		**10**
5		**10**
6		**10**
Total		**55**

Problems:

1. *[10 points]* Let $\vec{F}$ and $\vec{G}$ be two vector fields on $\mathbb{R}^3$. Denote by $M_{\vec{F}\cdot\vec{G}}$ the mass form associated with $\vec{F}\cdot\vec{G}$, denote by $W_{\vec{F}}$ the work form associated with $\vec{F}$ and denote by $\Phi_{\vec{G}}$ the flux form associated with $\vec{G}$. Show that
$$M_{\vec{F}\cdot\vec{G}} = W_{\vec{F}} \wedge \Phi_{\vec{G}}.$$

2. *[10 points]*
   (a) Let φ be a 1-form on $\mathbb{R}^2$, let ψ be a 2-form on $\mathbb{R}^2$ and assume $\varphi \neq 0$. Show that there is a 1-form, ω, on $\mathbb{R}^2$ such that $\varphi \wedge \omega = \psi$.
   (b) Now, let φ be a 1-form on $\mathbb{R}^3$, let ψ be a 2-form on $\mathbb{R}^3$ and assume $\varphi \neq 0$. In general, is there a 1-form, ω, on $\mathbb{R}^3$ such that $\varphi \wedge \omega = \psi$? Explain your answer to receive any credit.

3. *[5 points]* Consider the manifold $\mathcal{M}$ in $\mathbb{R}^4$ given by $x_1^2 + x_2^2 = 1$ and $x_3^2 + x_4^2 = 1$. Find the surface area of $\mathcal{M}$.

4. *[10 points]* Let $\mathbf{c}$ be the curve parametrized by $\gamma(t) = (t, 2t, t^2)$ where $t \in [0, 2]$ and let $\mathbf{c}$ be oriented by $\gamma'(t)$. Let $\vec{\mathbf{F}}$ be a force given by
$$\vec{F}(x, y, z) = (e^{2x+3y}, xe^{x^2}, 1).$$
Calculate the work done by $\vec{\mathbf{F}}$ along $\mathbf{c}$.

5. *[10 points]* Let S be the surface described by the graph of the hyperbolic paraboloid $z = y^2 - x^2$, above the region $D : x^2 + y^2 \leq 1$. Orient S by the upward pointing normal (i.e., the normal with positive z-component) and let $\vec{\mathbf{F}}$ be the vector field given by $F(x, y, z) = (y, x, 1)$. Compute the flux of $\vec{\mathbf{F}}$ through the surface.

6. *[10 points]* Consider the manifold $\mathcal{M} \subset \mathbb{R}^n$, given by $\sum_{i=1}^{n-1} x_i^2 = x_n$ and consider the function f given by $f(x_1, ..., x_n) := \sum_{i=1}^{n-1} x_i^2 - x_n$. Let Ω be the orientation of $\mathcal{M}$ by the gradient of f.

   (a) Write an expression for $\Omega(\vec{\mathbf{v}}_1, ..., \vec{\mathbf{v}}_{n-1})$.

   (b) Find a *direct* basis for the tangent space to $\mathcal{M}$ at the point $(x_1, ..., x_n)$ where $x_1 = x_n = 1$, and $x_i = 0$ for $i = 2, ..., n-1$.