MATH 31CH, **Exam 1** **April 21, 2015**

Name and PID: ______________________________

Instructions:

- Write your Name and PID.
- **You may not use any electronic devices, textbooks, or notes.** If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped.
- You must justify your work (legibly) to receive credit.
- **You may NOT use homework problems (without proof) in your solutions.**

Good Luck!

Problem	**Score**	**Out of**
1		**5**
2		**10**
3		**10**
4		**10**
5		**10**
6		**10**
Total		**55**

Problems:

1. *[5 points]* Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)dxdy$$

by first changing the order of integration.

2. *[10 points]*

   (a) Let D be the region in $\mathbb{R}^3$ inside the paraboloid $z = x^2 + y^2$, above the plane $z = 1$, and below the plane $z = 4$. Compute the volume of D.

   (b) Let W be the region in $\mathbb{R}^3$ between the half-cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2+y^2+z^2 = 1$. Compute the volume of W.

3. *[10 points]* Use an appropriate change of variables to calculate

$$\int\int_R xy\ dxdy$$

over the region R bounded by the curves of $xy = 1$, $xy = 3$, $y = x^2$, and $y = 3x^2$.

4. *[10 points]* Let P be a 3-parallelogram in $\mathbb{R}^4$ with sides $\vec{v}_1, \vec{v}_2, \vec{v}_3$ emanating from the same vertex. Suppose that $\vec{v}_1, \vec{v}_2, \vec{v}_3$ have lengths $1, 2,$ and 3 respectively, and that the angle between any pair of them is $\pi/3$. Find the 3-dimensional volume of P.

5. *[10 points]* Let A be an $n \times n$ matrix and let $\lambda_1, ..., \lambda_n$ be its (not necessarily distinct) eigenvalues. Prove that

$$\det(A) = \prod_{i=1}^{n} \lambda_i.$$

You may use the fact that any polynomial $p(z) = z^n + c_{n-1}z^{n-1} + ... + c_0$ can be factorized as $p(z) = (z - z_1)(z - z_2)...(z - z_n)$ where $z_1, ...z_n$ are the zeros of the polynomial.

6. *[10 points]* For any 2×2 matrices A, B, C, D denote by M the 4×4 matrix given by

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}.$$

Now, let Δ be the function given by $\Delta(M) = \det A \det D - \det B \det C$. Show that $\Delta(M)$ is NOT $\det M$.