1. Some definitions and formulas

Definition 1.1. *Suppose that $M \subseteq \mathbb{R}^n$ is a k-dimensional manifold. Let $\gamma : U \to \mathbb{R}^n$ be a relaxed parametrization of M, where X is the set of bad points. Then*

$$\operatorname{vol}_k M = \int_{U-X} \sqrt{\det([D\gamma(\vec{u})]^T [D\gamma(\vec{u})])} |d^k \vec{u}|.$$

Definition 1.2. *let $U \subseteq \mathbb{R}^k$ be a bounded open set with $\operatorname{vol}_k \partial U = 0$. Let V be an open subset of $\mathbb{R}^n$ and let $\gamma : U \to \mathbb{R}^n$ be a C^1 mapping with $\gamma(U) \subseteq V$. Let φ be a k-form field defined on V. Then the integral of φ over $[\gamma(U)]$ is*

$$\int_{[\gamma(U)]} \varphi = \int_U \varphi(P_{\gamma(\vec{u})}(\vec{D}_1\gamma(\vec{u}), \ldots, \vec{D}_k\gamma(\vec{u}))) |d^k \vec{u}|.$$

Definition 1.3. *Let $M \subseteq \mathbb{R}^n$ be a k-dimensional oriented manifold, φ a k-form field on a neighborhood of M, and $\gamma : U \to \mathbb{R}^n$ an orientation-preserving parametrization of M. Then*

$$\int_M \varphi = \int_{[\gamma(U)]} \varphi$$

as defined in the previous definition.