Math 31CH Spring 2017 Homework 6, due 5/17/2017 in HW box in the basement of AP&M by 5 pm

1 Upcoming exam

The midterm exam will cover up through Section 6.4, concentrating on material from Homeworks 4-6 and Sections 5.3, 6.1-6.4.

2 Exercises to submit on Wednesday 5/17

2.1 Exercises from the text

Section 6.3: #3, 4, 6, 8, 11
Section 6.4: #1, 4, 7

3 Exercises not from the text to submit on Wednesday 5/17

1. Let $M \in \mathbb{R}^n$ be a k-dimensional manifold. Suppose there is a continuous function $f : M \to (\mathbb{R}^n)^k$, where we write $f(\vec{x}) = (\vec{v}_1(\vec{x}), \vec{v}_2(\vec{x}), \ldots, \vec{v}_k(\vec{x}))$. Assume that for each $\vec{x}$, $\{\vec{v}_1(\vec{x}), \ldots, \vec{v}_k(\vec{x})\}$ is a basis for $T_{\vec{x}}(M)$. Show that there is a unique orientation of M such that for each $\vec{x} \in M$, the ordered basis $\{\vec{v}_1(\vec{x}), \ldots, \vec{v}_k(\vec{x})\}$ of $T_{\vec{x}}(M)$ is direct, in other words gets assigned the value 1.

2. Let $M \in \mathbb{R}^n$ be a k-dimensional manifold. Suppose that $\gamma : U \to \mathbb{R}^n$ is a parametrization of M in the sense of Definition 3.1.18 in the text. Note this is not a "relaxed" parametrization as in Definition 5.2.3, but it is equivalent to a relaxed parametrization in which the set X of bad points is empty. Assume further that the function $\gamma^{-1} : M \to U$ is continuous.

Show that there is a unique orientation of M such that for each $\vec{u} \in U$, the ordered basis $\vec{D}_1\gamma(\vec{u}), \ldots, \vec{D}_k\gamma(\vec{u})$ is direct in $T_{\gamma(\vec{u})}(M)$.