Math 31CH Spring 2017 Homework 5, due 5/10/2017 in HW box in the basement of AP&M by 5 pm

1 Reading

Read Sections 6.3-6.4.

2 Exercises to submit on Wednesday 5/10

2.1 Exercises from the text

Section 6.1: #3, 8, 10, 12

Section 6.2: #1, 2, 3

3 Exercise not from the text to submit on Wednesday 5/10

Recall that we defined the wedge product of forms differently from the text. Namely if $\varphi = dx_{i_1} \wedge \cdots \wedge dx_{i_k}$ is an elementary k-form on $\mathbb{R}^n$ and $\omega = dx_{j_1} \wedge \cdots \wedge dx_{j_l}$ is an elementary l-form on $\mathbb{R}^n$, then we put $\varphi \wedge \omega = dx_{i_1} \wedge \cdots \wedge dx_{i_k} \wedge dx_{j_1} \wedge \cdots \wedge dx_{j_l}$. (This may not be elementary, but it is either 0 or after reordering some of the terms is plus or minus an elementary $k + l$ form on $\mathbb{R}^n$.)

Then since $A_c^k(\mathbb{R}^n)$ has the elementary k-forms as a basis and $A_c^l(\mathbb{R}^n)$ has the elementary l-forms as a basis (as we proved), to define the wedge product $\varphi \wedge \omega$ for an arbitrary $\varphi \in A_c^k(\mathbb{R}^n)$ and $\omega \in A_c^l(\mathbb{R}^n)$ we "extend linearly". In other words, if $t_1, \ldots, t_K$ is the basis of $A_c^k(\mathbb{R}^n)$ given by elementary k-forms, and if $u_1, \ldots, u_L$ is the basis of $A_c^l(\mathbb{R}^n)$ given by elementary l-forms, then we define

$$(\sum_i a_i t_i) \wedge (\sum_j b_j u_j) = \sum_{i,j} a_i b_j (t_i \wedge u_j).$$

1. Prove Proposition 6.1.15 in the text, using our definition of wedge product. To prove associativity, for example, one possible strategy is to first prove that if associativity holds for wedge products of basis elements (elementary forms), then it holds in general. A similar strategy helps to prove the other properties.

2 (optional problem). Prove that our definition of wedge product of forms is the same as Definition 6.1.12 in the book (which is defined in a completely different way).