Math 31CH Spring 2017 Homework 1, due 4/12/2017 in HW box in the basement of AP&M by 5 pm

1 Reading

Read Sections 4.1, 4.5, 4.8, 4.9.

2 Exercises to submit on Wednesday 4/12

2.1 Exercises from the text

Section 4.1: #4, 9, 21

(Hint for #21(b): can the functions f and g be integrable? If not, what examples of non-integrable functions do you know?)

Section 4.5: # 7, 12, 15, 16

2.2 Exercises not from the text

1. Fix any real numbers a and b with $a \neq b$. Let $\mathbb{Q}$ be the rational numbers. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} a & x \in \mathbb{Q} \\ b & x \notin \mathbb{Q} \end{cases}$$

Let $A = [0, 1]$ be the closed interval. Show that the function $g(x) = 1_A(x)f(x)$ is bounded and has bounded support, but that g is not integrable. (Hint: You may use without proof the fact that for every pair of real numbers $c < d$, the open interval (c, d) contains both rational and irrational numbers. I'm not sure if this standard result was proved earlier in this course.)

2. Let A be the region of $\mathbb{R}^2$ defined by $\{(x, y)|0 \leq x, y \leq 1\}$, i.e. the unit square with side length 1.

Show directly from the definition of the integral that $f(x, y) = (1_A)(x^2 + y)$ is integrable, and calculate $\int_{\mathbb{R}^2} f|d^2\vec{x}|$ directly from the definition. Do not use any other properties of integrals such as the theorem that the integral of a sum is the sum of the integrals.

When you are done, check your answer by calculating $\int_{\mathbb{R}^2} f|d^2\vec{x}|$ using Fubini's theorem.

You may use in your proof the formulas

$$\sum_{i=1}^{n} i = n(n+1)/2$$

and

$$\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6.$$