

Math 207b Spring 2011: An introduction to quiver representations

MWF 12-12:50, 5402 AP&M
Professor D. Rogalski

This course will be an introduction to the language and theory of quivers and their representations. This subject plays an important role in a number of current research topics in noncommutative algebra. It also has connections to a number of other subjects, for example algebraic geometry. The background necessary for the first part of this course is roughly Math 200, though concurrent enrollment in Math 200 is possible.

In the first part of the course (more than half of the quarter), I will cover the basic theory of finite dimensional algebras, quivers and their path algebras, and representations. The goal will be to prove Gabriel's beautiful theorem characterizing path algebras of finite representation type as those associated to quivers of type A, D, or E. Other generally useful topics I may touch on along the way include Morita equivalence, projective resolutions, global dimension, and root systems.

In the second part I will discuss some further topics related to quivers; one possible topic is to talk a bit about the McKay correspondence. In the second part I may use some algebraic geometry; but I will attempt to make the course as self-contained as possible.

1. SOME REFERENCES

We do not plan to follow any text closely, but here is a list of other sources that cover the basic material we will cover:

1. "Elements of the Representation Theory of Associative Algebras", by I. Assem, D. Simson, A. Skowronski. I have the library copy, which I need as a reference; so please don't recall it. This book is a very comprehensive introduction to the theory of modules over finite-dimensional algebras.

2. Some course notes of Harm Derksen can be found at <http://www.math.lsa.umich.edu/~hderksen/math711.w01/math711.html>. These are a bit rough but the first 5 lectures give a fairly self-contained presentation of the material needed for Gabriel's theorem. I followed these notes more closely the last time I taught this course.

3. A more polished (but also more advanced) introduction is "Lectures on Representations of Quivers" by Bill Crawley-Boevey which can be found at

<http://www.amsta.leeds.ac.uk/~pmtwc/>

4. There are several other interesting papers on Bill Crawley-Boevey's website, for example "Representations of quivers, preprojective algebras and deformations of quotient singularities." This paper gives some more recent results connecting quivers and resolutions of singularities in algebraic geometry.

2. COURSE MECHANICS

I will occasionally post homework problems on the course website (which will be up and running later this week). I will also leave some things as exercises in the lectures; going over your lecture notes and doing these exercises is an important part of getting something lasting out of this course. I am happy to discuss these homework problems with you in office hours. There will be no formal requirement to write up and hand in solutions; though you are welcome to if you want to. There will be no exams.

I will not have formal office hours specifically for this course. Feel free to just stop by; or make an appointment to see me.