Math 200b Winter 2011 Midterm

February 9, 2011

Do as many of the problems as well as you can; you are not necessarily expected to finish all problems. You may quote results proved in class or in the textbook, but not if they trivialize the problem. Also, avoid quoting results proved only in homework exercises.

NAME:

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

1. Let $A = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \in M_2(\mathbb{C})$ where a, b, d are arbitrary complex numbers. Find with proof the Jordan canonical form of A, in terms of a, b, d.

2. Let M be a left module over an integral domain R. Recall that $m \in M$ is a torsion element if there exists $0 \neq a \in R$ such that $am = 0$. Recall that a module M is torsionfree if 0 is the only torsion element in M.

Let $\operatorname{Tor}(M)$ be the set of torsion elements in M. Prove that $\operatorname{Tor}(M)$ is a submodule of M. Then prove that the factor module $M/\operatorname{Tor}(M)$ is a torsionfree module.

3. Let K be the splitting field of $f(x) = x^4 - 2$ over $\mathbb{Q}$. Determine $[K : \mathbb{Q}]$ with proof.

4. Let $F \subseteq K$ be a field extension where K is the splitting field of some irreducible polynomial $f \in F[x]$. Suppose that $\alpha_1, \alpha_2 \in K$ are both roots of f. Prove, quoting basic results we covered, that there exists an isomorphism $\sigma : K \to K$ such that $\sigma(\alpha_1) = \alpha_2$. (Make sure you make clear what the hypotheses are of the results you are using and that these hypotheses are satisfied.)

Also, give an example which shows that this result does not hold in general for non-irreducible polynomials f.