MATH 200B WINTER 2022 MIDTERM

Instructions:

You may quote major theorems proved in class or the notes, but not if the whole point of the problem is to reproduce the proof of such a theorem. Similarly, quote the result of a homework exercise only if the result of the exercise is a fundamental fact and reproducing the result of the exercise is not the main point of the problem.

You may use the result of one part of the problem in the proof of a later part, even you do not complete the earlier part.

1 (15 pts). Let $A \in M_3(F)$ where F is an algebraically closed field. Classify up to similarity the matrices A which satisfy $A^3 = A$, by describing their possible Jordan forms. There may be cases depending on the characteristic of F. Make sure you justify your answer.

2 (15 pts). Let R be a PID and M a nonzero finitely generated torsion left R-module. Recall that M is *uniserial* if there is a finite chain of submodules

$$M_0 = \{0\} \subseteq M_1 \subseteq \cdots \subseteq M_n = M$$

such that the M_i are all of the R-submodules of M.

(a) (10 pts) Show that M has only one elementary divisor if and only if M is uniserial.

(b) (5 pts) Show that M has only one invariant factor if and only if M is cyclic.

Date: February 7, 2021.