MATH 200B WINTER 2020 MIDTERM

Instructions: Show all of your work. You may use the result of one part of a problem in the proof of a later part, even if you do not complete the proof of the earlier part. You may quote basic theorems proved in the textbook or in class, unless the problem tells you not to, or unless the whole point of the problem is to reproduce the proof of such a basic theorem.

1. Recall that a module M is *indecomposable* if it cannot be written as an internal direct sum $M = N_1 \bigoplus N_2$ for nonzero submodules N_1 and N_2. Let M be a nonzero finitely generated torsion module over a PID R. Show that M is indecomposable if and only if it has a single elementary divisor.

2. Let S be a commutative ring and let R be a unital subring of S. Let $M_2(S)$ be the ring of 2×2-matrices with coefficients in S. Prove that

$$M_2(R) \otimes_R S \cong M_2(S)$$

as S-algebras.

3. Let F be a field. A matrix $A \in M_n(F)$ is a *projection matrix* if $A^2 = A$. Show that there are precisely $n + 1$ similarity classes of projection matrices.

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